Structure and Hierarchy in Real-Time Systems

Modeling and Analysis

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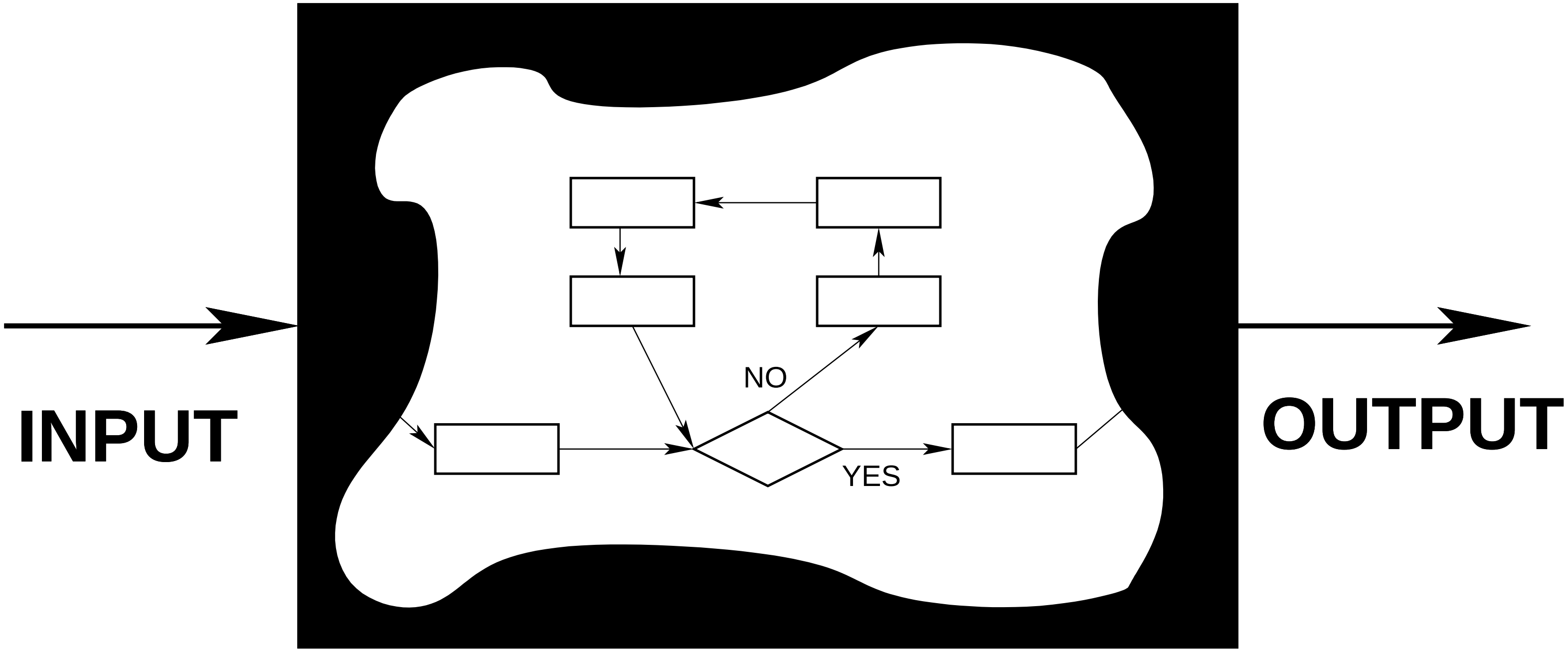
Traditional Input/Output Programs

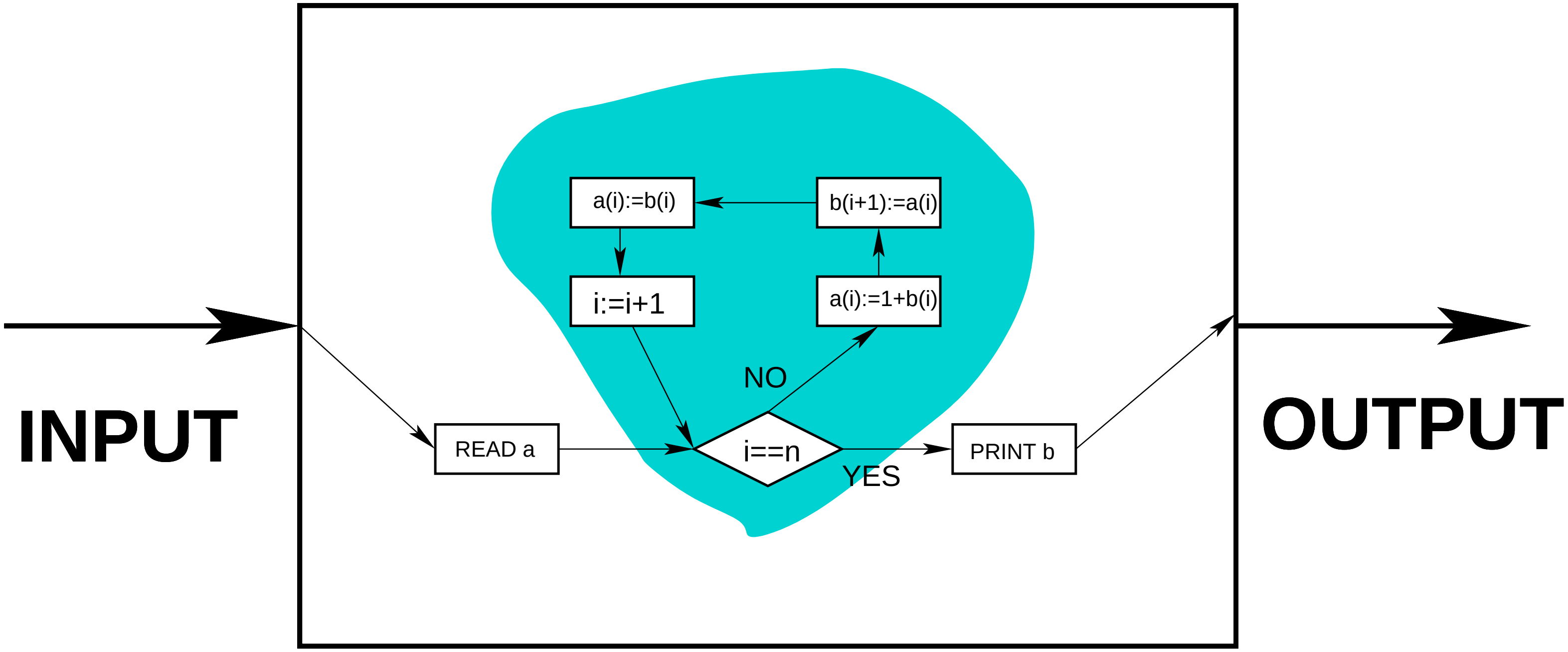
Correctness $:=$ relation over **Input** / **Output**

Traditional Input/Output Programs



Correctness	:=	relation over **Input** / **Output**
Testing	:=	try some **typical** and some **borderline** cases

Traditional Input/Output Programs



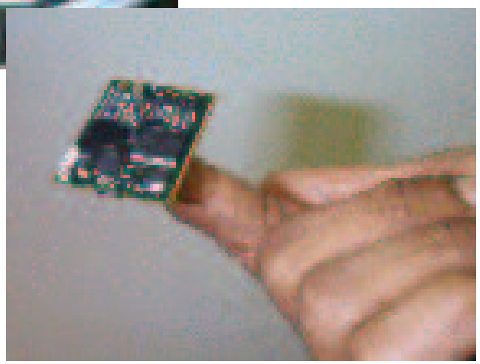
Correctness := relation over **Input** / **Output**

Testing := try some **typical** and some **borderline** cases

Analysis := proof something for **ALL** inputs

can use **assertions** on sub-structures

Application: Reactive Systems (?)



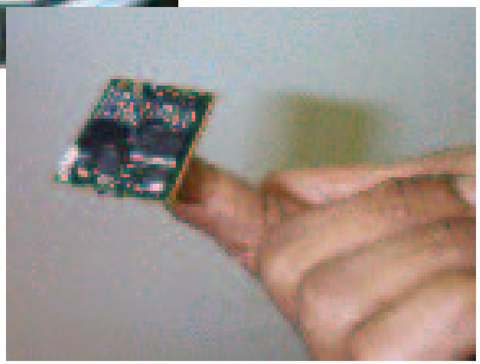
Embedded:

mixture of hard- and software;

severe resource limitations;

interaction with environment

Application: Reactive Systems (?)



Embedded:

mixture of hard- and software;
severe resource limitations;
interaction with environment

Real-Time:

Correctness not only dependent on the logical order of events, but also on their timing

Reactive Systems are Different

black cactus

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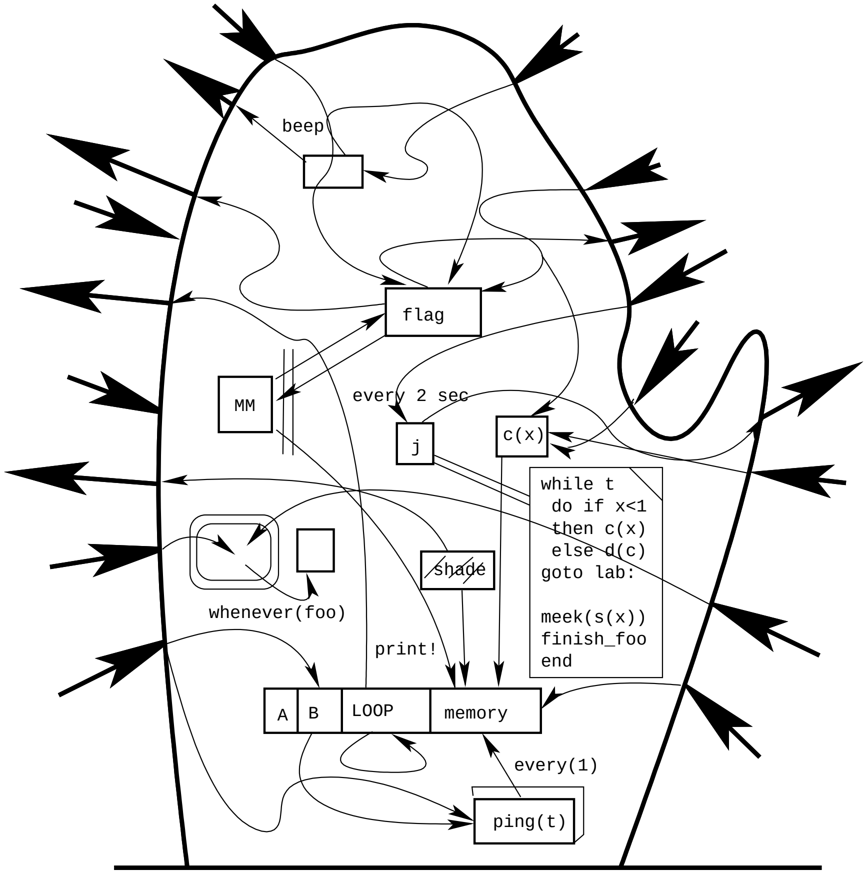
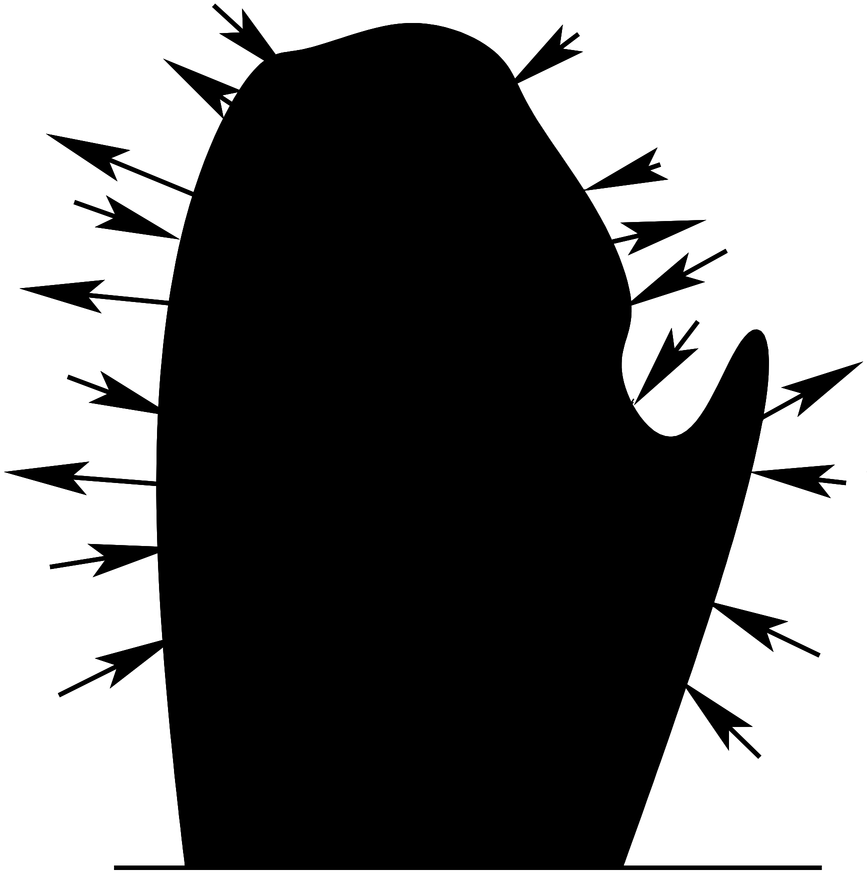


black cactus

messy implementation

Reactive Systems are Different



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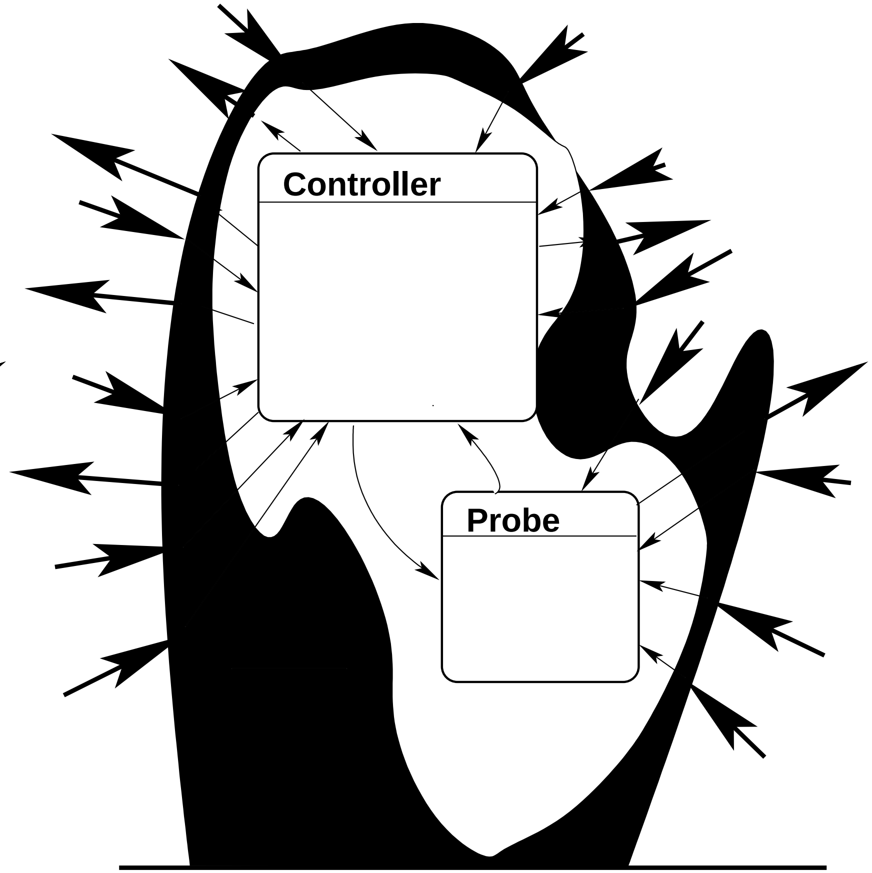
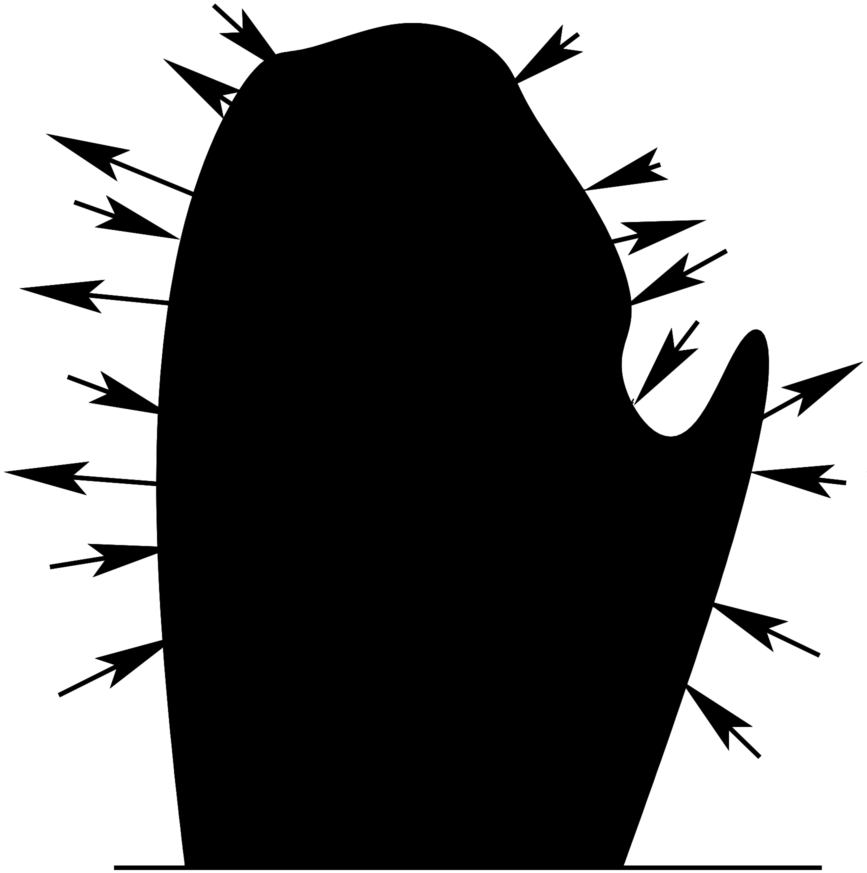
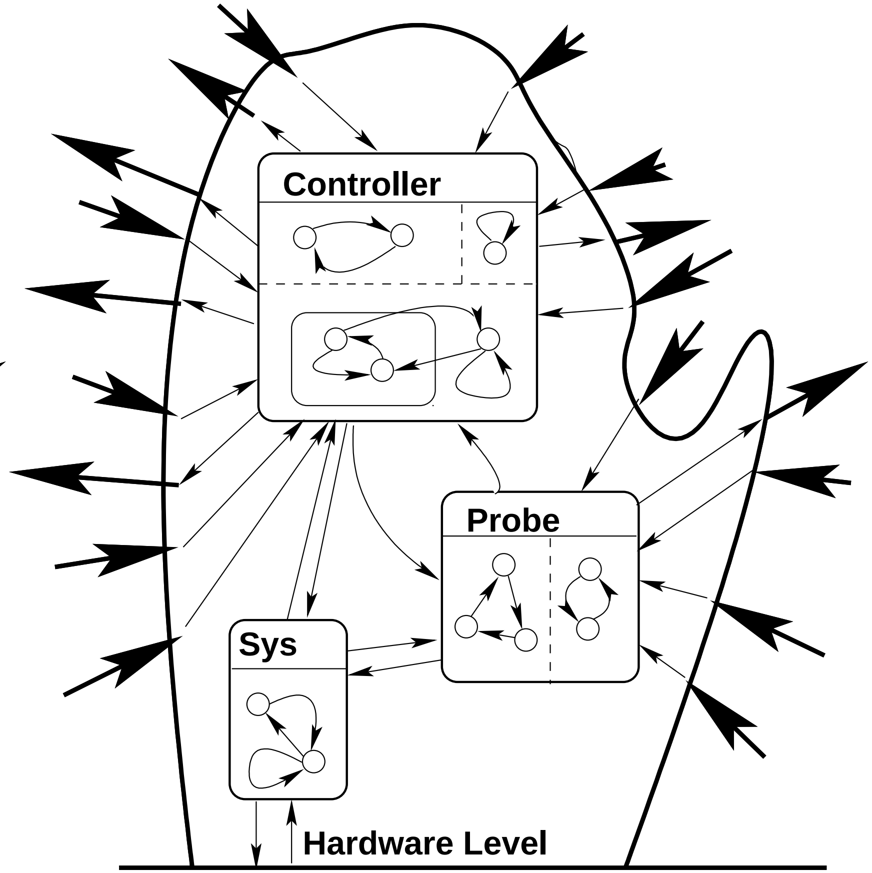


structured

Reactive Systems are Different

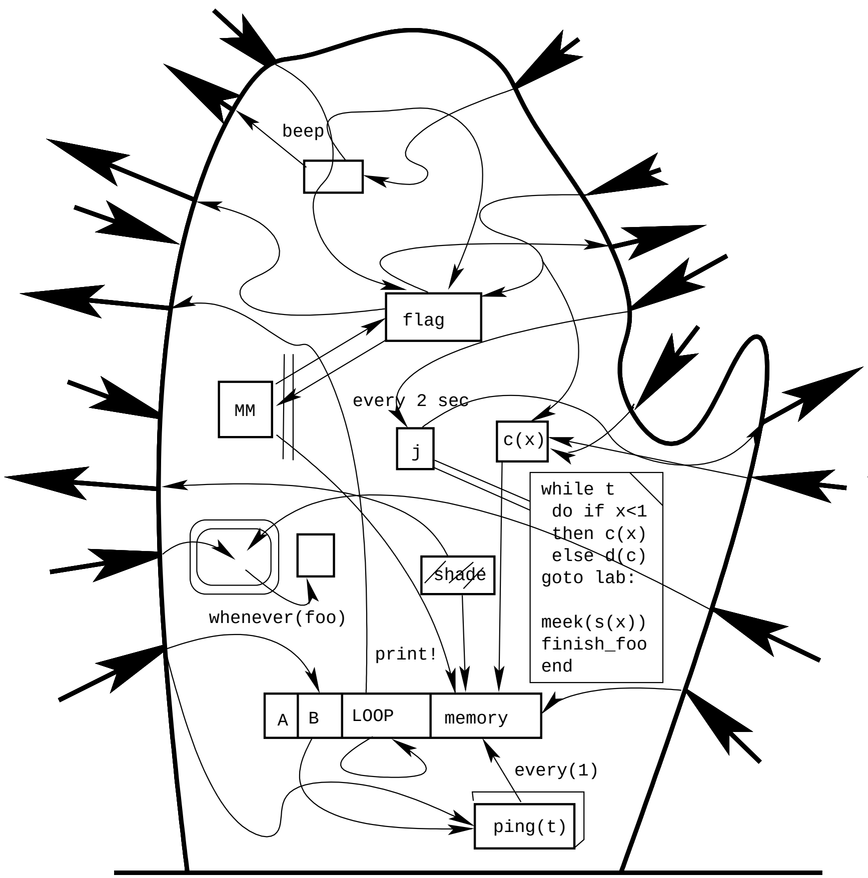


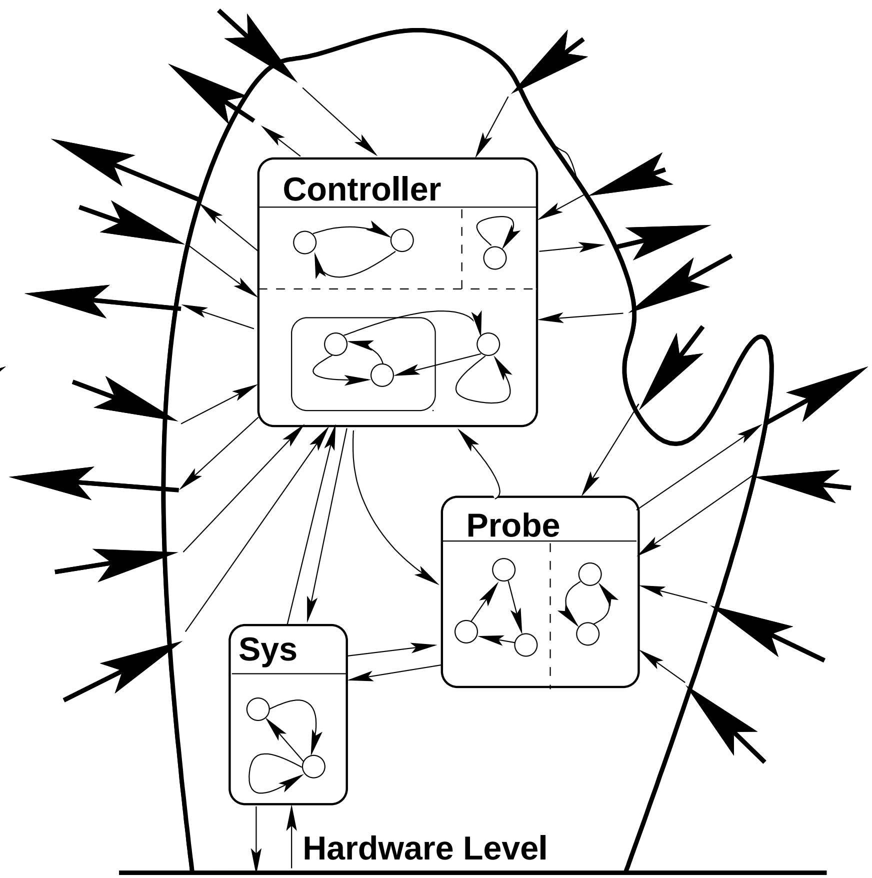


black cactus | **structured**

messy implementation | **different levels**

Reactive Systems are Different





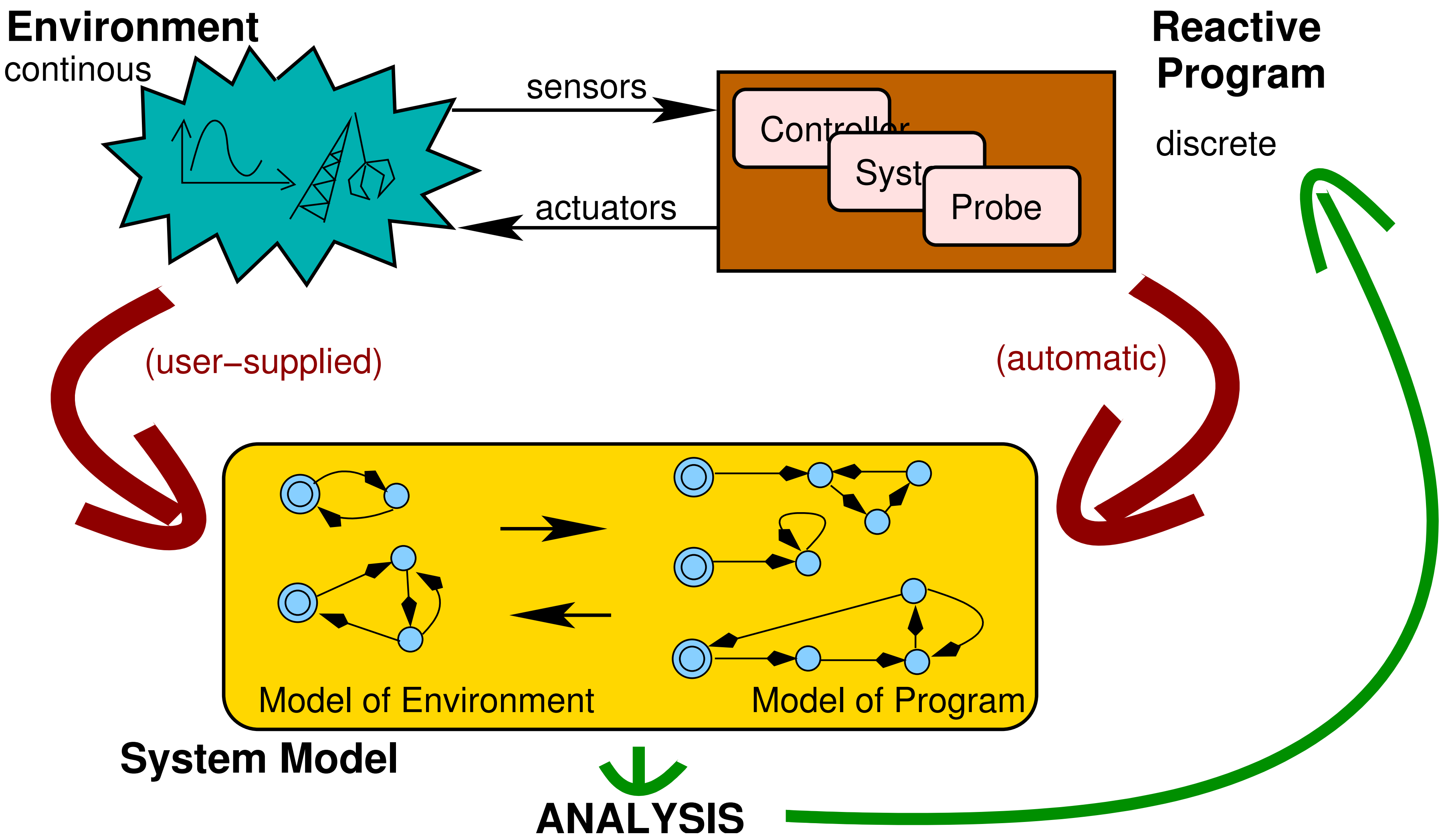
black cactus

messy implementation

structured

different levels

Composing the Embedded System Model



The Big Questions

- What are appropriate **languages** to model a reactive system **?**
- How do we perform **analysis** on partially completed systems **?**

Outline of the Thesis

Part I: Modeling of Real-Time Systems

1. The unified modeling language (UML) and statecharts (overview)
2. The language of UPPAAL (trace-based semantics)
3. Hierarchical timed automata [NWPT'01,FASE'02,journal submission]

Part II: Algorithmic Verification of Real-Time Systems

4. Real-time model checking: forward analysis (correctness formalization)
5. Optimization techniques for real-time systems (benchmarks)
6. Model augmentation to speed-up model checking [TPTS'01]
7. Predicate abstraction for dense real-time [TPTS'01]

Part III: Making Use of Hierarchical Structure

8. Construction of good hierarchies from parallel components [CHARME'01]
9. Flattening hierarchical timed automata for model checking [NWPT'01,FASE'02,journal submission]

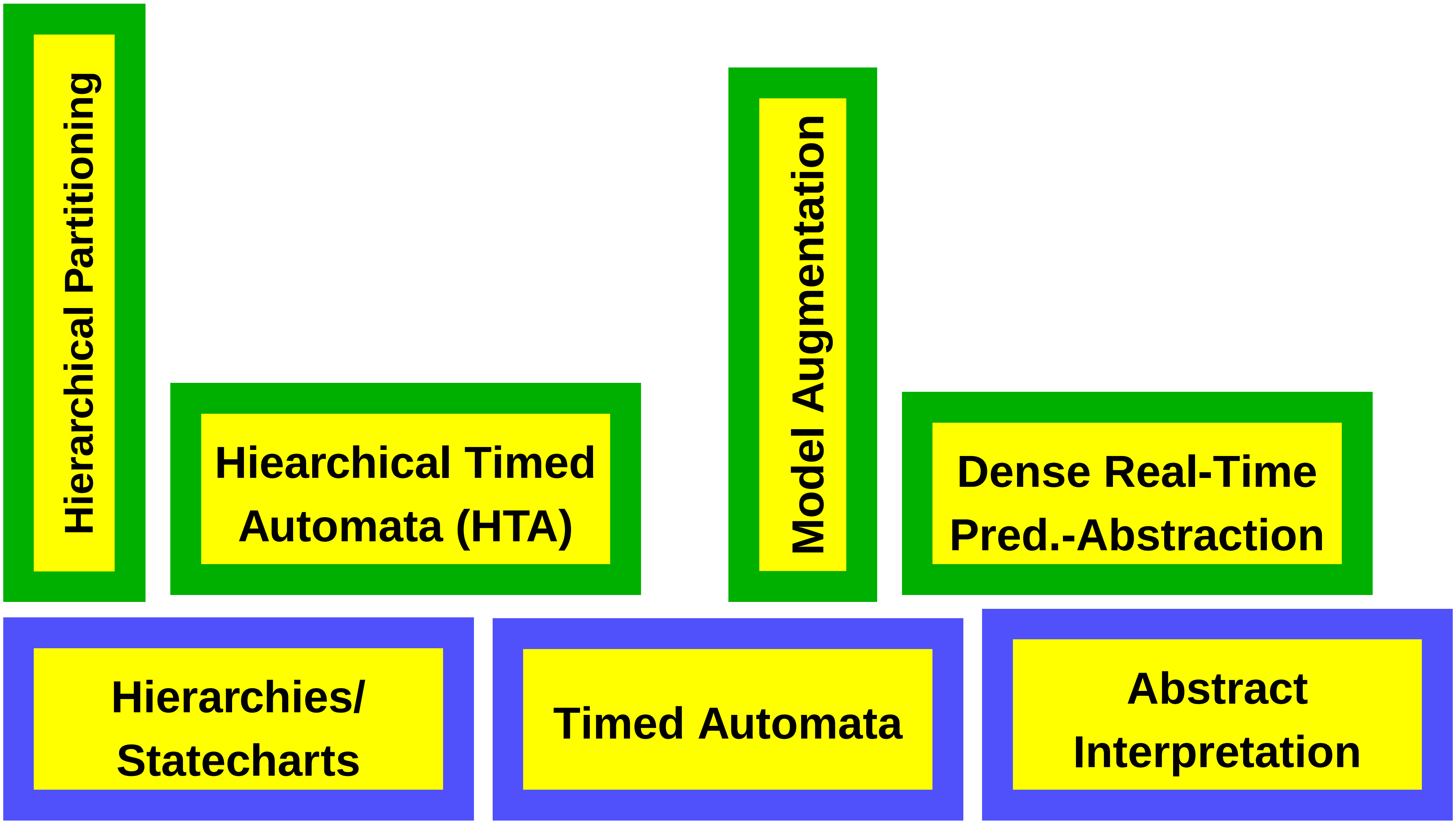
What was Known, What is New?

Hierarchies/ Statecharts

Timed Automata

Abstract Interpretation

What was Known, What is New?



Outline of the Thesis – and this Talk

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In the Following

Chapter 3: Hierarchical Timed Automata

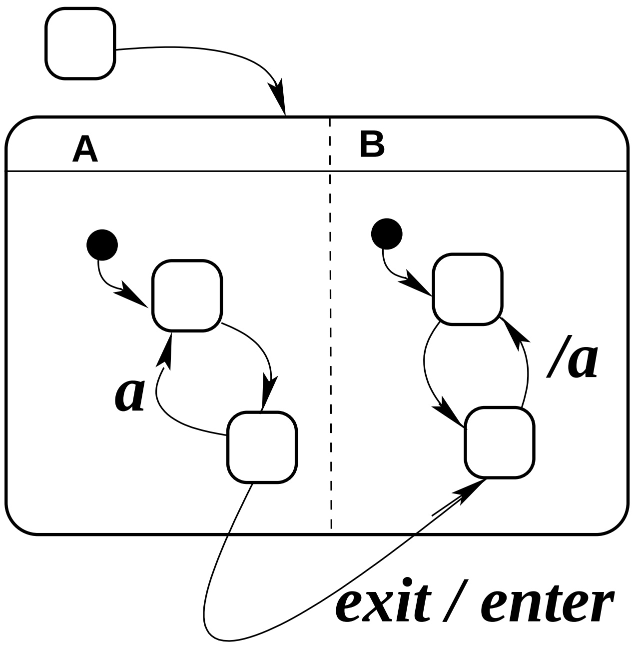
1. Restricted Statecharts with Real-Time
2. A Trace-Based Semantics
3. Flattening and Correspondence
4. Case Study: Cardiac Pacemaker

Chapter 7: Predicate Abstraction for Dense Real-Time

5. Galois Connection to an Untimed Model
6. Progress Assumption by Restricting Delays
7. Successive Refinement

Summary

The Statechart Formalism



Features

- hierarchical state machines
- parallelism (on any level)
- history
- event communication
- powerful synchronization mechanisms
- inter-level transitions
- actions that are dependent on states
- actions on entry/exit
- ...

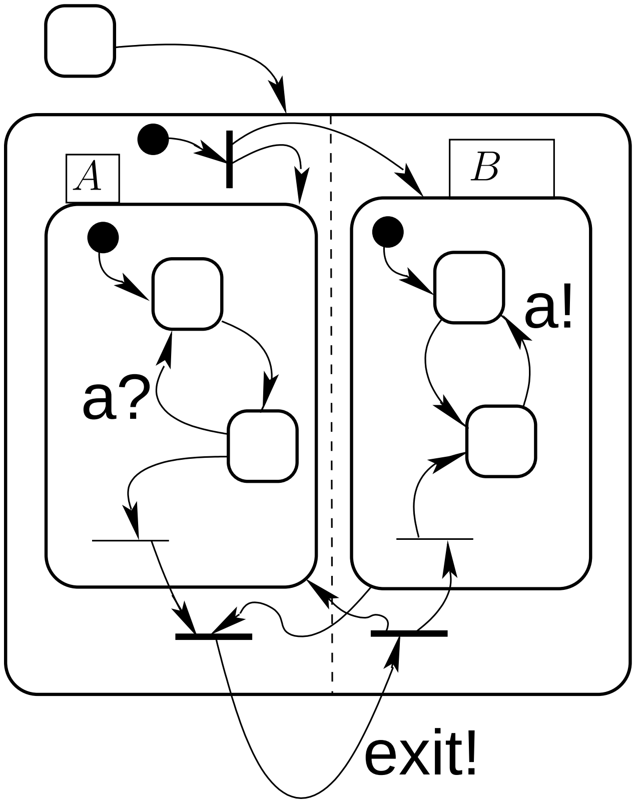
Claim:

The statechart formalism is *appropriate* for the development of reactive systems

Fact:

Basic statechart properties are undecidable
$\Rightarrow$ automated analysis *impossible* in general

Restricted Statechart Formalism



Concentration on key features

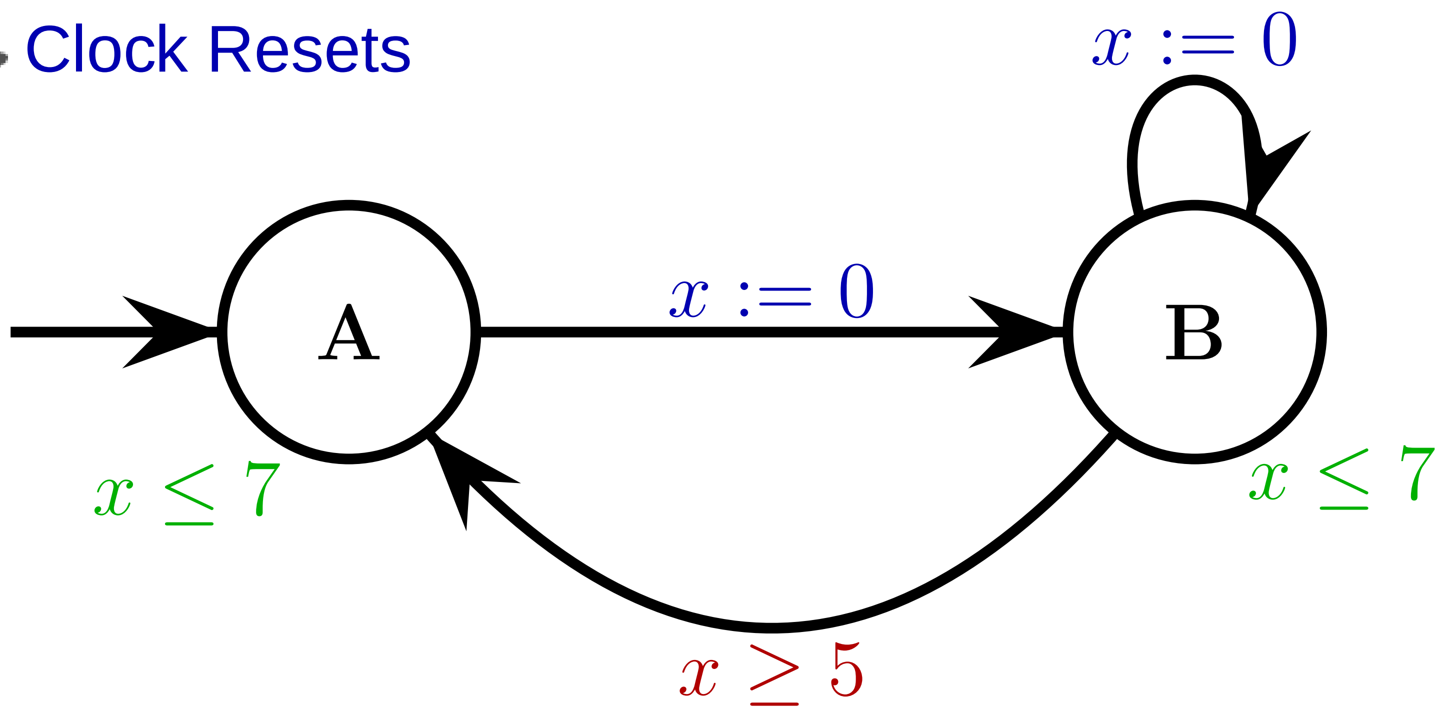
- hierarchical state machines ✔
- parallelism (on any level) ✔
- history ✔
- **no** event communication
- **no** sync states
- **no** inter-level transitions
- **no** actions that are dependent on states
- **no** actions on entry/exit

instead:

- hand-shake style synchronization
- shared variables

Real-Time Extensions

- Clocks
- (timed) Guards
- Invariants
- Clock Resets



Hierarchical Timed Automata (HTAs)

The HTA formalizm has a formal semantics

- operational style
- interprets time as ”dense real-time”
- trace-based

Benefits

- unambiguous
- mechanizable
- you can proof something about it

Semantic Rules (Example)

configuration: $\langle \rho,\ \mu,\ \nu,\ \theta \rangle$ with

ρ : control locations

μ : valuation of integer variables

ν : valuation of clocks

θ : history

operation:

$t : l \xrightarrow{g,s,r,u} l', \rho, \mu, \nu$ a transition

$$\frac{g(\mu,\nu) \quad \textit{JoinEnabled}(\rho,\mu,\nu,l) \quad \textit{Inv}(\rho^{\mathcal{T}_t},\nu^{\mathcal{T}_t}) \quad \neg\textit{EXIT}(l')}{(\rho,\mu,\nu,\theta) \xrightarrow{t} \mathcal{T}_t(\rho,\mu,\nu,\theta)}\ \textit{action}$$

Ingredients for the Semantic Rules

$$\textit{JoinEnabled}(\rho,\mu,\nu,S) := \textit{BASIC}(S) \vee \exists E \in \textit{PreExitSets}(S).\ \forall b \in \textit{Leaves}(\rho,S).\ \exists b' \in E.\ b \xrightarrow{g} b' \ \wedge\ g(\mu,\nu)$$

$$\textit{PreExitSets}(l) := \begin{cases} \left.\begin{array}{l} \displaystyle\bigcup_{n_1,\ldots,n_k} \boxtimes_{1\le i\le k} \textit{PreExitSets}(n_i), \text{ where} \\ \quad k = |\tilde{\delta}(\delta^{-1}(l))|,\ \{n_1,\ldots,n_k\} \subseteq \delta^{\times}(\delta^{-1}(l)), \\ \quad \forall i.\textit{EXIT}(n_i)\ \wedge\ n_i \to l \in T \\ \quad \{\delta^{-1}(n_1),\ldots,\delta^{-1}(n_k)\} = \tilde{\delta}(l) \end{array}\right\} & \text{if } \begin{array}{l}\textit{EXIT}(l)\wedge \\ \textit{AND}(\delta^{-1}(l))\end{array} \\ \left.\begin{array}{l} \displaystyle\bigcup_{m\in\delta(\delta^{-1}(l))} \textit{PreExitSets}(m), \text{ where } m \xrightarrow{g,r} l \in T \\ \quad \cup \{\{l\}\} \end{array}\right\} & \text{if } \begin{array}{l}\textit{EXIT}(l)\wedge \\ \textit{XOR}(\delta^{-1}(l))\end{array} \\ \{\} & \text{if } \textit{BASIC}(l) \end{cases}$$

This Formalizm is

1) hierarchical

2) timed

3) decidable

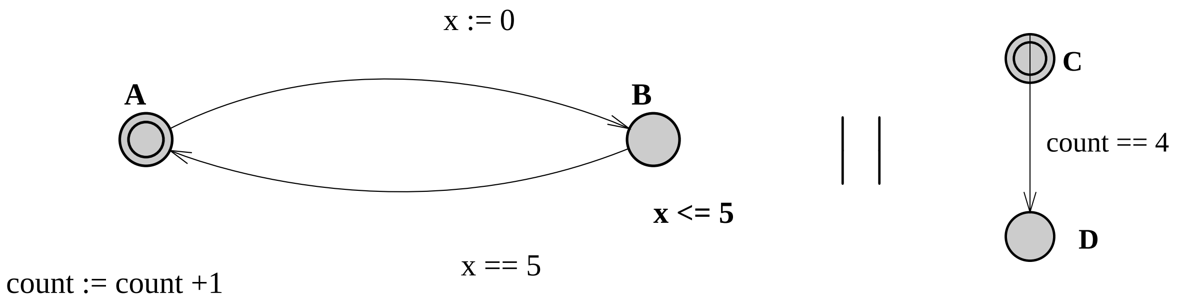
Model Checking

$$\boxed{M \overset{?}{\models} \varphi}$$

M	:	description of the system
φ	:	desired (correctness) property

- easier than proving a general theorem
- completely automatic ('yes' or counterexample)
- *efficient* algorithms tailored for classes of problems

Real-Time Model Checking with UPPAAL



network of (flat) timed automata

`clock x; int count`

Only a subset of timed computation tree logic (TCTL) supported:

`E<>` φ	reachability
`A[]` φ	safety (invariantly φ)
`E[]` φ	possibly always φ
`A<>` φ	inevitably φ
`A[]` $\varphi \Rightarrow$ `A<>` ψ	unbounded response

φ, ψ : propositional formula over locations and (existing) clocks

Outline of the Flattening

Basically:

one superstate $\longrightarrow$ one (parallel) automaton

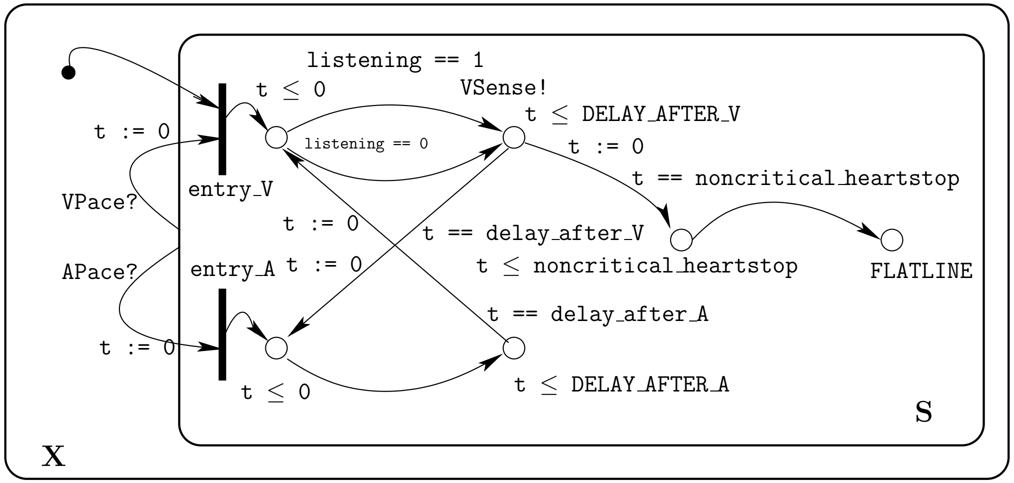
$+$ some housekeeping

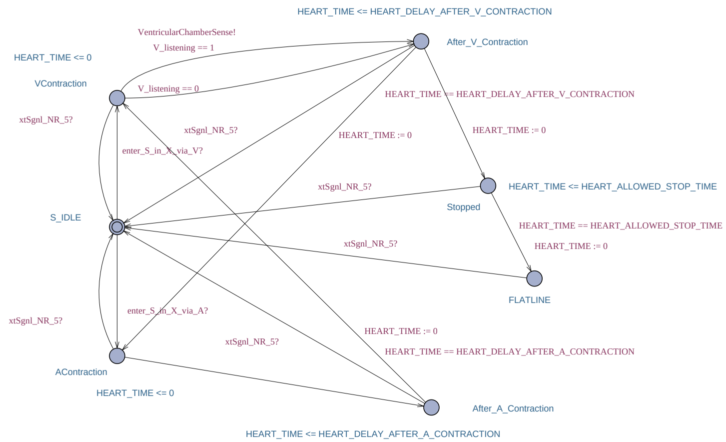
Problems:

- template mechanism
- scope of channels
- pre-compuation of all possible global joins

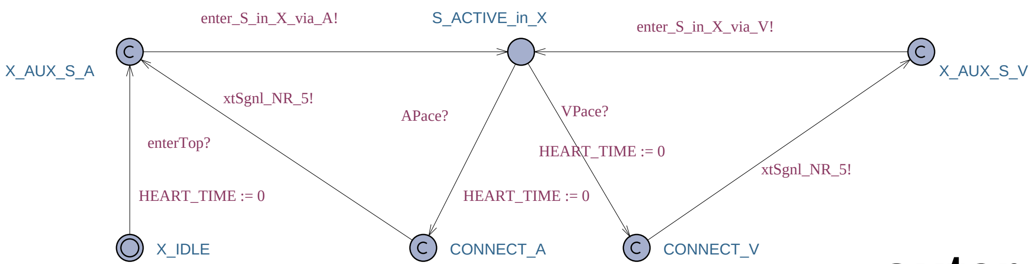
$\approx$ **10˙000 lines of documented Java code**

Example: Flattening the Model of a Human Heart





inner superstate



outer superstate

Soundness & Correctness

Translations introduce slack. Thus

$$\boxed{M_H} \models \varphi \quad \not\Leftrightarrow \quad \boxed{\textit{flatten}(M_H)} \models \textit{flatten}(\varphi)$$

but

$$M_H \models \varphi \qquad \Leftrightarrow \qquad \textit{flatten}(M_H) \models_{project(M)} \textit{flatten}(\varphi)$$

timed transition system		timed *flatten*(M) traces
$\downarrow$ give rise to		$\downarrow$ project to M_H
timed M_H traces	*match*	timed M_H traces

From (timed) Statecharts to UPPAAL

Rhapsody timed Statechart	$\longrightarrow$	*HTA model* $\boxed{M_H}$	$\longrightarrow$	*TA model* $\boxed{\textit{flatten}(M_H)}$
hierarchical model		TA-close hierarchy		MODEL-CHECK
informal description		formal semantics		formal semantics

From (timed) Statecharts to UPPAAL

Rhapsody timed Statechart $\longrightarrow$ *HTA model* $\boxed{M_H}$ $\longrightarrow$ *TA model* $\boxed{\textit{flatten}(M_H)}$

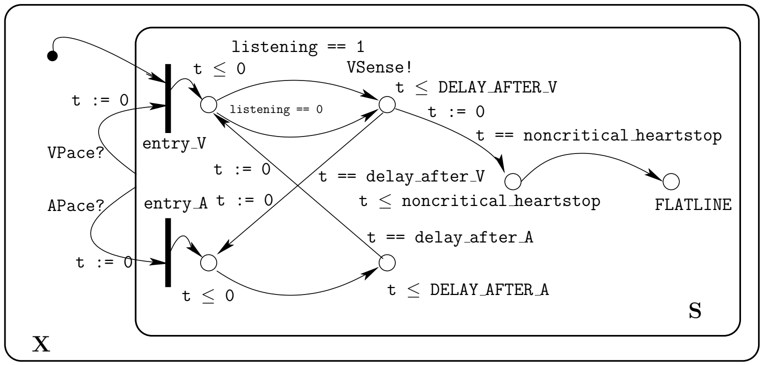
Rhapsody timed Statechart	*HTA model* M_H	*TA model* *flatten*(M_H)
hierarchical model	TA-close hierarchy	MODEL-CHECK
informal description	formal semantics	formal semantics

NORMALIZATION
simplification of data
(safe) omission of **C**$^{++}$ code

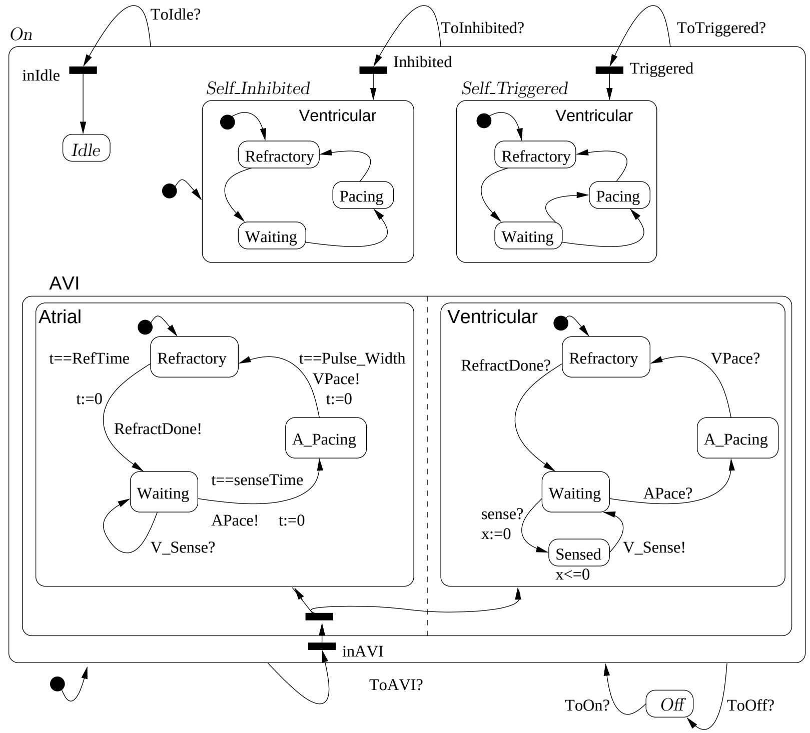
FLATTENING
auxiliary locations
auxiliary variables

Guiding Principle: *Make it easy to adjust to small changes!*

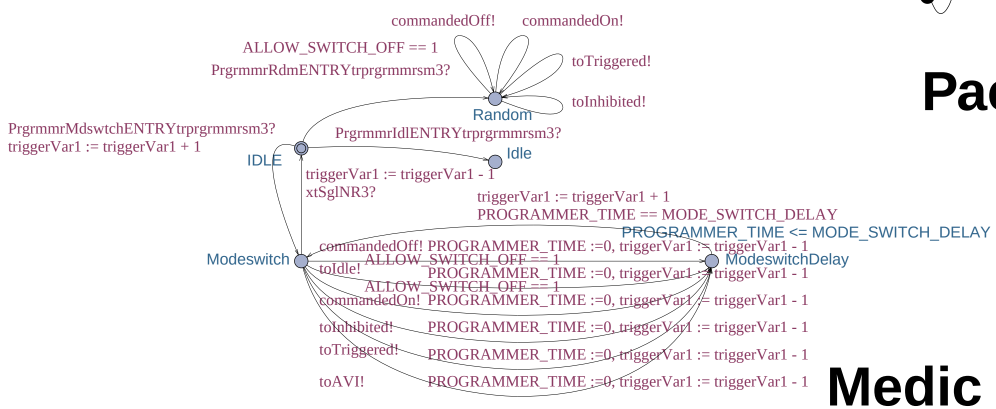
Model Checking a Pacemaker



Human Heart



Pacemaker



Medic

Model-Checking the Pacemaker

- DEADLOCK:
  possible (if heart stops)
- SAFETY:
  `A[] ¬heart stops`
  only true for 'good' medic
- LIVENESS:
  ```
  A[] ( Vcontract
        => A<> Acontract )
  ```

Model-Checking the Pacemaker

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  possible (if heart stops)
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  only true for 'good' medic
- LIVENESS:
  ```
  A[] ( Vcontract
        => A<> Acontract )
  ```

Parameters:

```
REFRACTORY_TIME  = 50
SENSE_TIMEOUT    = 15

DELAY_AFTER_V = 50
DELAY_AFTER_A =  5

HEART_ALLOWED_STOP_TIME = 135

MODE_SWITCH_DELAY  = 66
```

E.g. for `MODE_SWITCH_DELAY  = 65`, `A[] ¬heart stops` is violated

Summary on Hierarchical Timed Automata

The Major Gains:

- for **modeler**:
  more flexible and compact modeling (than with flat automata)
- for **development process**:
  intermediate format for automated analysis of design models
  (requires typically an abstraction step)

Future Work:

- put to use in AIT-WOODDES project
  RHAPSODY UML statecharts to be model-checked via UPPAAL
- to be integrated in the UPPAAL tool
  - → UPPAAL timed automata are a special case of HTAs
  - → editor for XML grammar is work in progress
  - → model checking engine for HTAs planned

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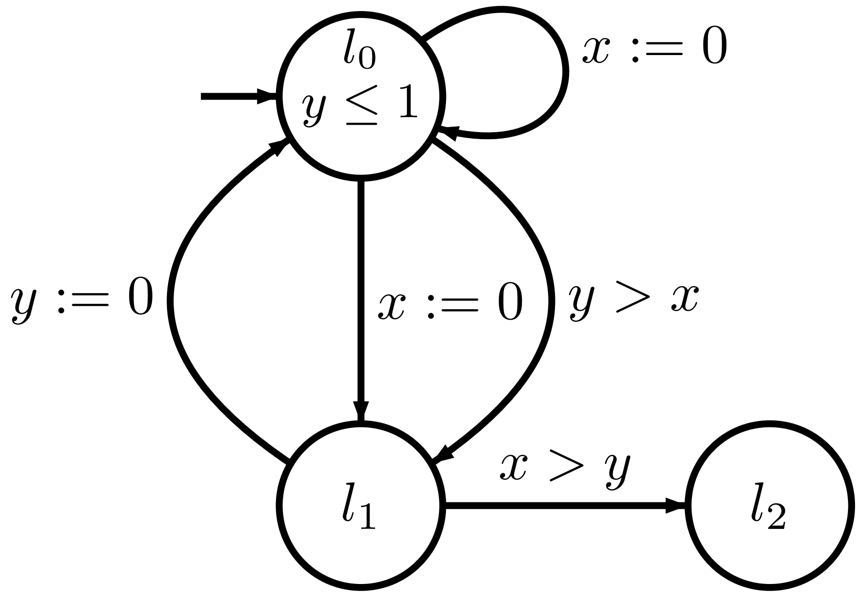
Summary

Timed Systems

Timing constraints Γ, propositional Symbols A

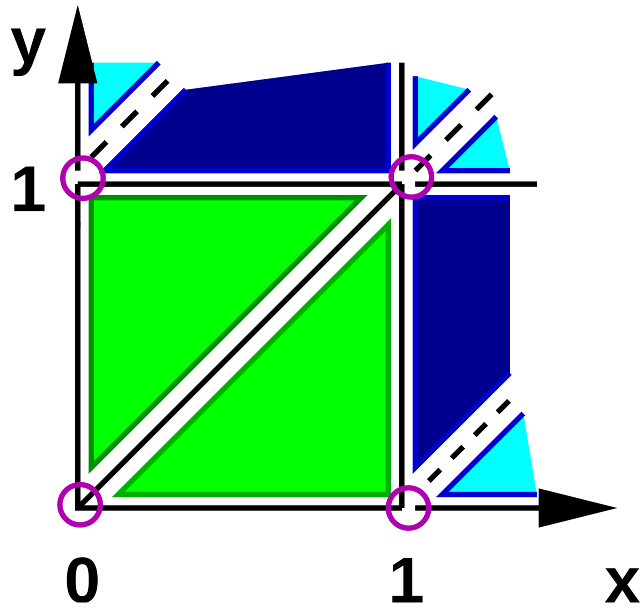
Timed System $\mathcal{S} = \langle L, P, C, \rightarrow, l_0, I \rangle$



Semantics as transition system $\mathcal{M} = \langle L \times \mathcal{V}_C, P, \Rightarrow, (l_0, \nu_0) \rangle$ with *non-zenoness assumption:*

$\boxed{\text{if trace infinite, sum over all delays is } \infty}$

Clock Regions



- Given: $\mathcal{S}$, C, $\tilde{c}$
- Finite partition of the infinite state space
- Clock region: $\mathcal{X}C \subseteq \mathcal{V}_C$ s.t. for all $\chi \in \mathit{Constr}(c)$ and for any two ν, $\nu' \in \mathcal{X}C$ it is the case that $\nu \models\!\!\approx \chi$ if and only if $\nu' \models\!\!\approx \chi$
- $\nu_1 \equiv_{\mathcal{S}} \nu_2$

Propositional Next-Free μ-Calculus

Syntax:

$$\varphi := tt \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \exists\,(\varphi_1 U \varphi_2) \mid \forall\,(\varphi_1 U \varphi_2) \mid Z \mid \mu Z.\varphi$$

Semantics: $[\![\varphi]\!]^{\mathcal{M}}_{\vartheta}$ . . . set of states for which φ holds

Intuitively, an existential (strong) until formula $\exists\,(\varphi_1 U \varphi_2)$ holds in some states $\mathbf{s}$ iff φ_1 holds on some path from $\mathbf{s}$ until φ_2 holds.

$$[\![\exists\,(\varphi_1 U \varphi_2)]\!]^{\mathcal{M}}_{\vartheta} \stackrel{\text{def}}{=}$$

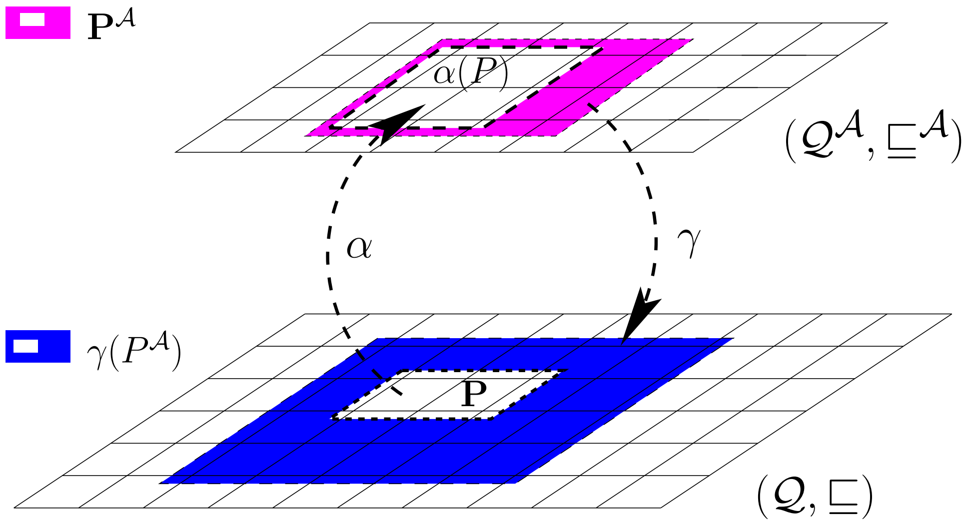
$$\{s_0 \in S \mid \text{there exists a path } \tau = (s_0 \Rightarrow s_1 \Rightarrow \ldots), \text{ s.t. } s_i \in [\![\varphi_2]\!]^{\mathcal{M}}_{\vartheta}$$

$$\text{for some } i \geq 0, \text{ and for all } 0 \leq j < i,\, s_j \in [\![\varphi_1]\!]^{\mathcal{M}}_{\vartheta}\}$$

State-Based Model Checking

- Semantics of formula φ $:=$ the set of configurations satisfying φ
- Model checking problem: $l_0 \overset{?}{\in} [\![\varphi]\!]^{\mathcal{M}} \rightarrow$ **Yes/No**
- Finite quotient for timed systems: *region construction*
- Our approach: successive refinements of finite *approximations*

Abstract Interpretation: Galois Connections



$(\mathcal{Q}^{\mathcal{A}}, \sqsubseteq^{\mathcal{A}})$ abstract system

$(\mathcal{Q}, \sqsubseteq)$ concrete system

$\alpha : \mathcal{Q} \rightarrow \mathcal{Q}^{\mathcal{A}}$ abstraction

$\gamma : \mathcal{Q}^{\mathcal{A}} \rightarrow \mathcal{Q}$ concretization

$$\boxed{\alpha(P) \sqsubseteq^{\mathcal{A}} P^{\mathcal{A}} \quad \Leftrightarrow \quad P \sqsubseteq \gamma(P^{\mathcal{A}})}$$

Essence: connection of 2 lattice structures

Problems: stability and self-loops

Predicate Abstraction of Timed Systems

Abstraction Predicates

- formula over clocks in C
  E.g.: $x - y \leq 3$, $x^2 - y^2 = 3.1415$,
- partition the (uncountable) state space with respect to their truth value
- set of abstractions predicates $\Psi = \{\psi_0, \ldots, \psi_{n-1}\}$

Abstraction function

- $\alpha : \quad \mathcal{V}_C \rightarrow \quad B_n$
- $\alpha(\ \nu)(i) := \quad \psi_i \nu$

Concretization function

- $\gamma : \quad B_n \rightarrow \quad \wp(\mathcal{V}_C)$
- $\gamma(\ b) := \{\nu \in \mathcal{V}_C \mid \bigwedge_{i=0}^{n-1} \psi_i \nu \equiv b(i) \quad \}$

Predicate Abstraction of Timed Systems

Abstraction Predicates

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- partition the (uncountable) state space with respect to their truth value
- set of abstractions predicates $\Psi = \{\psi_0, \ldots, \psi_{n-1}\}$

Abstraction function

- $\alpha : L \times \mathcal{V}_C \to L \times B_n$
- $\alpha(l,\nu)(i) := (l,\psi_i \nu)$

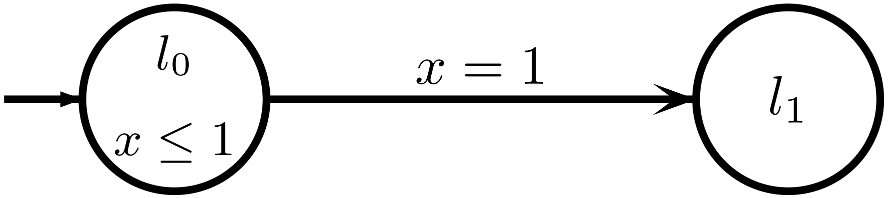
Concretization function

- $\gamma : L \times B_n \to L \times \wp(\mathcal{V}_C)$
- $\gamma(l,b) := \{\nu \in \mathcal{V}_C \mid \bigwedge_{i=0}^{n-1} \psi_i \nu \equiv b(i) \wedge I(l)\}$

Predicate Abstracted Semantics

$$
\begin{array}{rcl}
[\![tt]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}} & := & S^A \\
[\![p]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}} & := & \{(l,b) \in S^A \mid p \in P(l)\} \\
[\![\varphi_1 \wedge \varphi_2]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}} & := & [\![\varphi_1]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}} \cap [\![\varphi_2]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}} \\
[\![\neg\varphi]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}} & := & S^A \setminus [\![\varphi]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\bar{\sigma}}} \\
[\![\exists\, (\varphi_1 U \varphi_2)]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}} & := & \{s_0 \in S^A \mid \text{there exists a path } \tau = (s_0 \Rightarrow^{\sigma} s_1 \Rightarrow^{\sigma} s_1 \ldots), \\
 & & \text{s.t. } s_i \in [\![\varphi_2]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}} \text{ for some } i \geq 0, \text{ and} \\
 & & \text{for all } 0 \leq j < i,\ s_j \in [\![\varphi_1]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}} \\
[\![\forall\, (\varphi_1 U \varphi_2)]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}} & := & \{s_0 \in S^A \mid \text{for every path } \tau = (s_0 \Rightarrow^{\bar{\sigma}} s_1 \Rightarrow^{\bar{\sigma}} \ldots), \\
 & & \text{there exists } i \geq 0 \text{ s.t. } s_i \in [\![\varphi_2]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}}, \text{ and} \\
 & & \text{for all } 0 \leq j < i,\ s_j \in [\![\varphi_1]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}}\} \\
[\![Z]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}} & := & \vartheta(Z) \\
[\![\mu Z.\varphi]\!]_{\vartheta}^{\mathcal{M}_{\Psi}^{\sigma}} & := & \cap\{S' \in S^A \mid [\![\varphi]\!]_{\vartheta[Z:=S']}^{\mathcal{M}_{\Psi}^{\sigma}} \subseteq S'\}
\end{array}
$$

Example for Abstraction



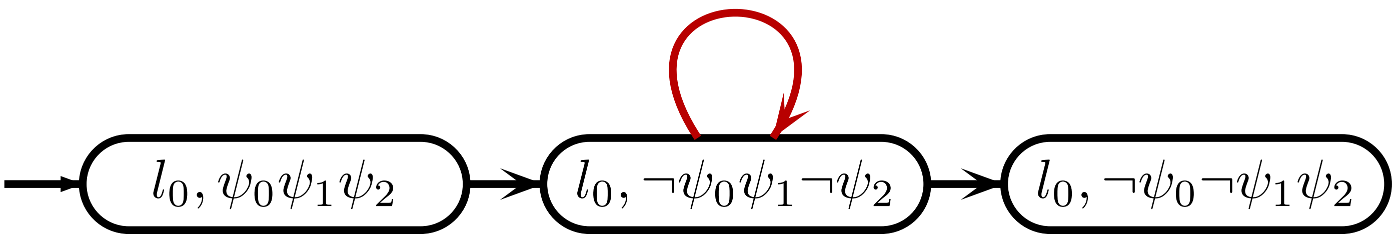
We want to verify: $\varphi = \forall\,(tt\;U\;at\_l_1)$

Abstraction predicates: $\{x = 0, x < 1, x = 1\}$

Assume the following sequence in the concrete trace:

$$(l_0, x = 0) \stackrel{1/2}{\Rightarrow} (l_0, x = 1/2) \stackrel{1/4}{\Rightarrow} (l_0, x = 3/4) \stackrel{1/4}{\Rightarrow} (l_0, x = 1) \stackrel{\mathbf{true}}{\Rightarrow} (l_1, x = 1)$$

Abstraction yields (only a fragment is illustrated):



Problem: spurious self-loop

Modified Semantics: Restricted Delay Step

Given: $\mathcal{S}$,C, $\tilde{c}$

A delay step $(l,\nu) \xrightarrow{\delta} (l,(\nu+\delta))$ is a restricted delay step iff it crosses the border of the current clock region:

$$\boxed{\exists x \in C.\, \exists k \in \{0,\ldots,c\}.\ \nu(x) = k \ \vee\ (\nu(x) < k \wedge \nu(x) + \delta \geq k)}$$

Restricted transition relation: $\Rightarrow_R \subseteq (L, \mathcal{V}_C) \times (L, \mathcal{V}_C)$

The second delay step in the previous trace is disallowed:

$$(l_0, x = 0) \Rightarrow_R (l_0, x = 1/2) \not\Rightarrow_R (l_0, x = 3/4) \Rightarrow_R (l_0, x = 1) \Rightarrow_R (l_1, x = 1)$$

Theorem:

$$[\![\varphi]\!]_\vartheta^{\mathcal{M}} \ = \ [\![\varphi]\!]_\vartheta^{\mathcal{M}_\mathcal{R}}$$

Abstraction is Sound & Complete

Given: $\mathcal{M} = \langle S^C, P, \Rightarrow, s_0^C \rangle$ a transition system

Ψ a set of predicates

$\mathcal{M}_\Psi^+, \mathcal{M}_\Psi^-$ the over-/under-approximations

Theorem: $\gamma(\llbracket\varphi\rrbracket^{\mathcal{M}_\Psi^-}) \subseteq \llbracket\varphi\rrbracket^{\mathcal{M}} \subseteq \gamma(\llbracket\varphi\rrbracket^{\mathcal{M}_\Psi^+})$

Theorem:

If $(\forall\psi \in \Psi.\ \psi\nu_1 \Leftrightarrow \psi\nu_2) \Rightarrow \nu_1 \equiv_{\mathcal{S}} \nu_2$

Then $\llbracket\varphi\rrbracket_\vartheta^{\mathcal{M}_\Psi^-} = \llbracket\varphi\rrbracket_\vartheta^{\mathcal{M}_\Psi^+}$

Refinement of the Abstraction

- Basis $\widehat{\Psi}$: the "exact" abstract transition system can be computed
  Not practicable
- Successive approximation of the abstract transition relation:

Algorithm: *refine_approximation*

INPUT $\mathcal{M}$, $\widehat{\Psi}$, φ

CHOOSE $\Psi \subseteq \widehat{\Psi}$

WHILE $l_0 \notin [\![\varphi]\!]^{\mathcal{M}^-_{\Psi}}$ /⋆ YES ⋆/

$\wedge \quad l_0 \notin [\![\neg\varphi]\!]^{\mathcal{M}^-_{\Psi}}$ /⋆ NO ⋆/

CHOOSE $\psi \in \widehat{\Psi} \setminus \Psi$

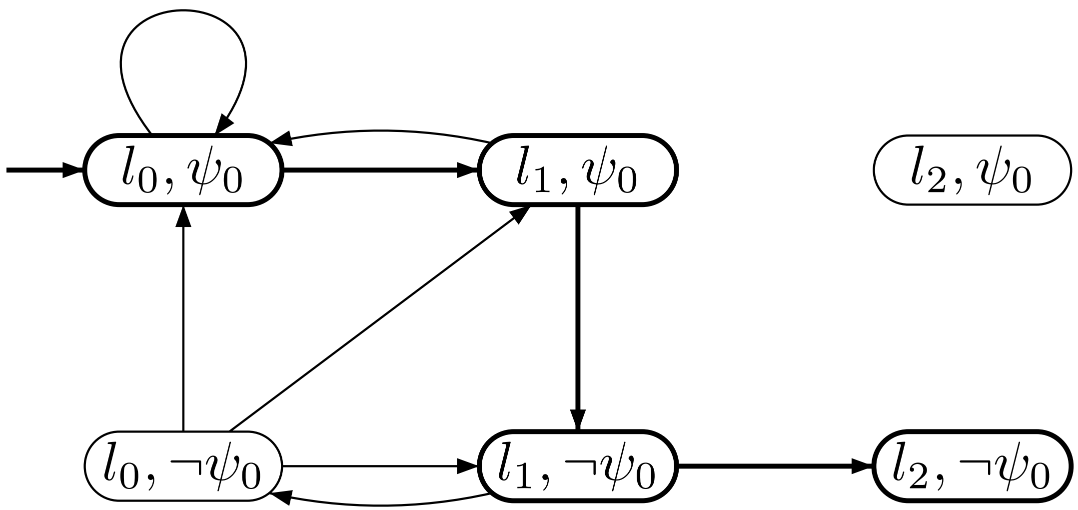
$\Psi := \Psi \cup \{\psi\}$

Example (Refinement)

$$\varphi := \neg\exists\,(tt\,U\,at\_l_2)$$

$$\Psi := \{x = 0, y = 0, x \leq 1, x \geq 1, y \leq 1, y \geq 1, x > y, x < y\}$$

I. $\psi_0 \equiv x = 0$



$\mathcal{M}^+_{\{x=0\}} \overset{?}{\models} \varphi$ NO

$$\tau = \Big(\ (l_0, \psi_0) \Rightarrow^+ (l_1, \psi_0) \Rightarrow^+ (l_1, \neg\psi_0) \Rightarrow^+ (l_2, \neg\psi_0)\ \Big)$$

Example – Continuation I.

$$\tau = (\underbrace{(l_0, \psi_0)}_{s_0} \Rightarrow^+ \underbrace{(l_1, \psi_0)}_{s_1} \Rightarrow^+ \underbrace{(l_1, \neg\psi_0)}_{s_2} \Rightarrow^+ \underbrace{(l_2, \neg\psi_0)}_{s_3})$$

Is there a corresponding counterexample on the concrete system?

$\exists\, \tau^c = (y_0 \Rightarrow y_1 \Rightarrow y_2 \Rightarrow y_3)$ s.t.

$y_0 \in \gamma(s_0),\ y_1 \in \gamma(s_1),\ y_2 \in \gamma(s_2),\ y_3 \in \gamma(s_3),\ y_0 = s_0^c$

$$\begin{aligned} F := \quad & y_0 \in \gamma(s_0) \ \wedge\ y_1 \in \gamma(s_1) \ \wedge\ y_2 \in \gamma(s_2) \ \wedge\ y_3 \in \gamma(s_3) \ \wedge \\ & y_1 \Rightarrow y_2 \ \wedge\ y_2 \Rightarrow y_3 \ \wedge\ y_0 = s_0^c \end{aligned}$$

Is F satisfiable?

Example – Continuation II.

Here F is unsatisfiable!

$$\begin{array}{lccc} y_0 \in & (l_0, x = y = 0) & \in \gamma(s_0) \\ & \Downarrow & \\ y_1 \in & (l_1, x = 0 \wedge 0 \leq y \leq 1) & \in \gamma(s_1) \\ & \Downarrow & \\ y_2 \in & (l_1, x > 0 \wedge y > x) & \in \gamma(s_2) \\ & \not\Downarrow & \\ y_3 \in & (l_1, x > 0 \wedge y \geq 0) & = \gamma(s_3) \end{array}$$

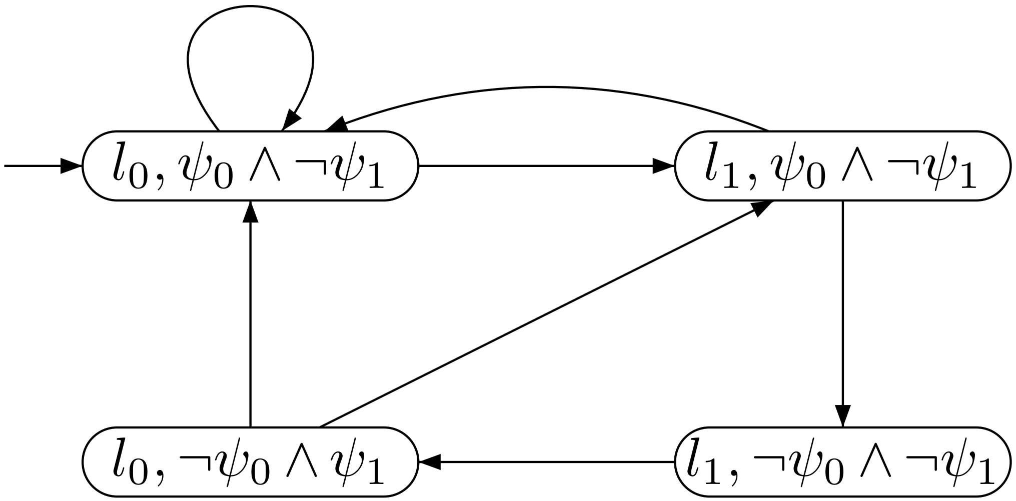
Choose $\psi_1 \in \Psi$ s.t. $\forall\, y \in \gamma(s_k),\ y' \in \gamma(s_{k+1}).\, y \not\Rightarrow y'$

Here: $k = 2$ $\psi_1 \equiv x > y$

Example – Continuation III.

New approximation $\mathcal{M}^{+}_{\{x=0, x>y\}}$

Satisfies formula $\varphi = \neg\exists\,(tt\,U\,at\_l_2)$



Algorithm terminates with **true**

$(l_0, x = y = 0) \;\in\; [\![\neg\exists\,(tt\,U\,at\_l_2)]\!]^{\mathcal{M}}$

Summary on Predicate Abstraction for Real-Time

What can be verified?

- Safety (known before)
- Liveness (!)

Observations:

- self-loops problem:
  solved by restricting the delay steps in *concrete* system
- logic is un-timed and *without next*
- a weaker assumption than non-zenoness suffices
  (only restrict infinite sequences of delay steps)

Summarizing...

Part I: Modeling of Real-Time Systems

1. The unified modeling language (UML) and statecharts (overview)
2. The language of UPPAAL (trace-based semantics)
3. Hierarchical timed automata [NWPT'01,FASE'02,journal submission]

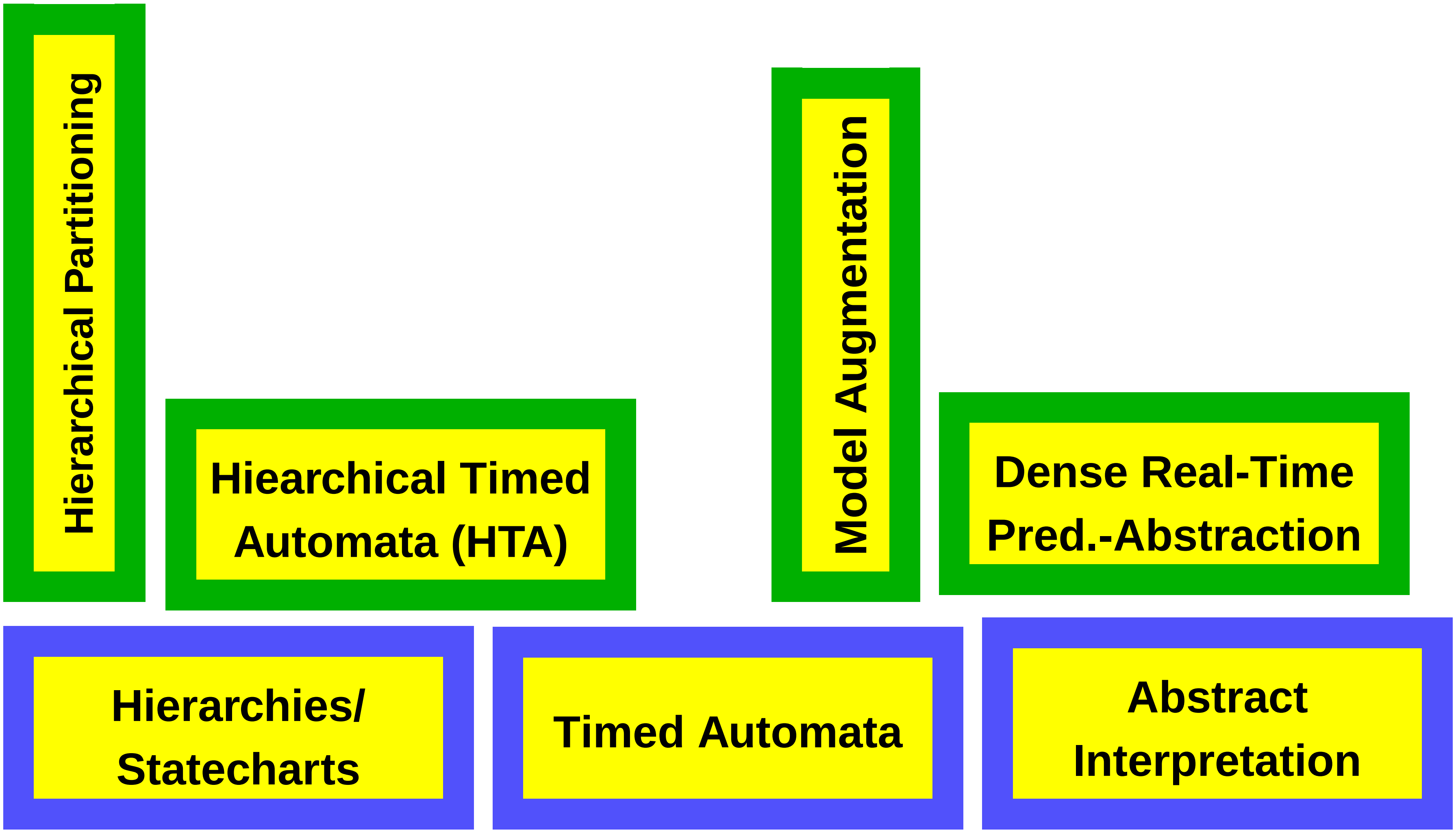
Part II: Algorithmic Verification of Real-Time Systems

4. Real-time model checking: forward analysis (correctness formalization)
5. Optimization techniques for real-time systems (benchmarks)
6. Model augmentation to speed-up model checking [TPTS'01]
7. Predicate abstraction for dense real-time [TPTS'01]

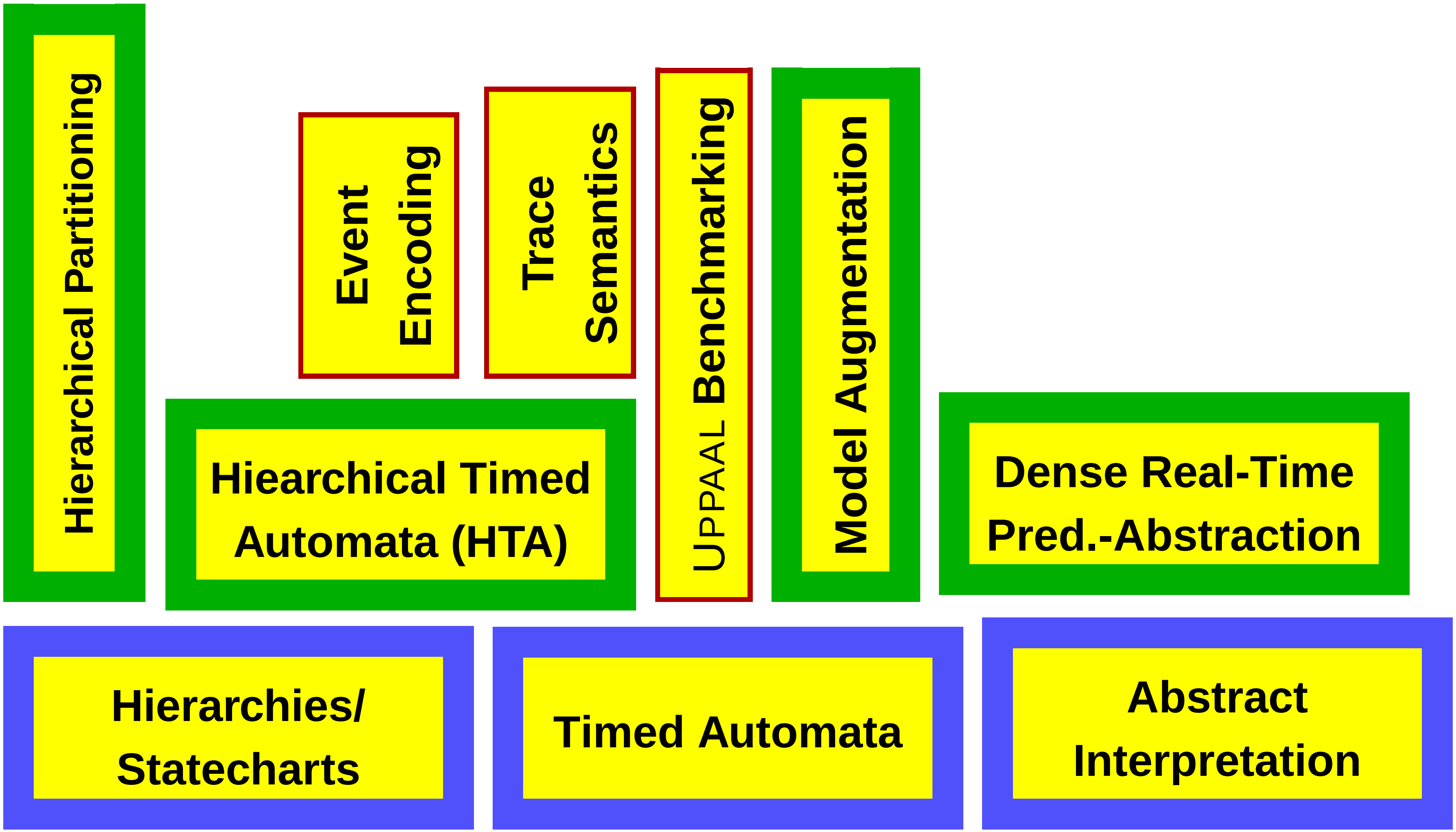
Part III: Making Use of Hierarchical Structure

8. Construction of good hierarchies from parallel components [CHARME'01]
9. Flattening hierarchical timed automata for model checking [NWPT'01,FASE'02,journal submission]

What was Known, What was New?



What was Known, What was New?



What is (still) to be Done

- Model checking **engine** for HTAs
  - → future work of Alexandre David, Uppsala University
- Exploiting HTAs via **re-use**
  - → similar to re-use in *modecharts* (Rajeev Alur et al.)
    similar to CBR technique in VISUALSTATE (Gerd Behrmann et al.)
    time gives rise to difficulties
- Sound abstraction step from **UML statecharts** to the HTA formalism
  - → Clearly requires *approximation* of data and events
- Implement the successive refinement idea for timed automata

Conclusion – on Real-Time Systems

- **Hierarchies** complicate—but do not hinder—the formal analysis; whether they also can be exploited remains to be seen
- Fully automated analysis is expensive but often **feasible** for reasonably sized models
  (which implies that formal methods should be applied *a priori*)
- Predominant efficiency gain is via *abstractions*;
  Techniques that **approximate** timed systems can go *beyond safety*

Outline of the Thesis

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