Category theory is a method to organize mathematics.

∅

Every equalizer is monic.

2

If f is an isomorphism, then the inverse f^{-1} is unique.

3

Terminal objects are unique up to isomorphism.

5

An epic equalizer is an isomorphism.

7

Limits are equalizers of products.

11

Equalizers - if existent - are determined up to isomorphism.

∞

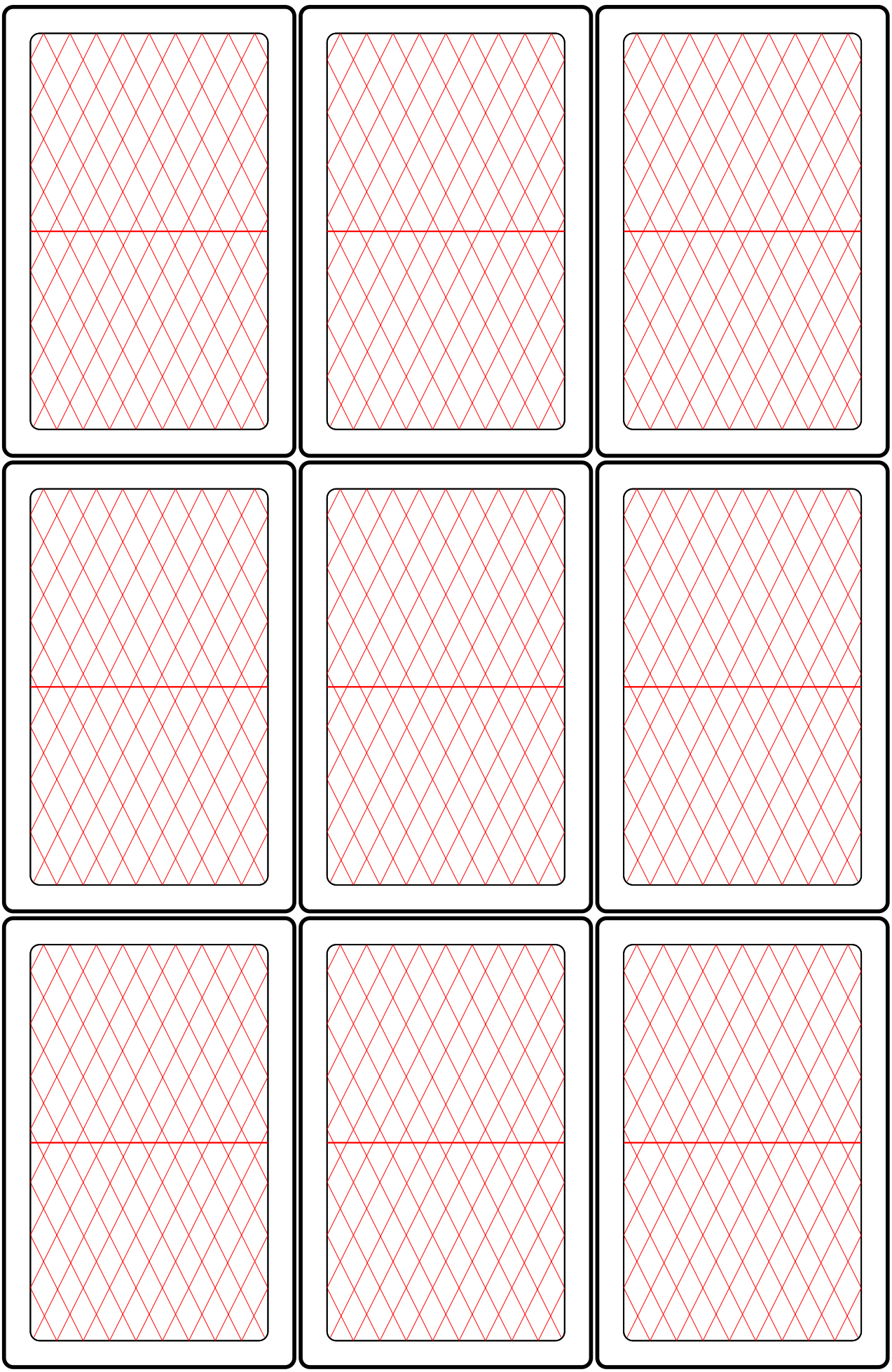
If $\mathcal{C}(X, Y)$ is always a set, $\mathcal{C}$ is locally small.

ω

A coequalizer provides the categorical analogue to the set-theoretic notion of equivalence classes.

$\aleph_\lambda$

CCARD V1.0



An arrow is fully determined by its effects on generalized elements.

∅

XM CCARD V1.0

An arrow is fully determined by its effects on generalized elements.

∅

Coequalizers cannot be defined in terms of general elements.

2

XM CCARD V1.0

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2

The cones for a diagram form a category; the limit is the terminal object of this.

3

XM CCARD V1.0

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3

If a category has a terminal object $\mathbf{1}$, every object is its own product with $\mathbf{1}$.

5

XM CCARD V1.0

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5

For A object of $\mathcal{C}$, $h : f \to g$ of $\mathcal{C}/A$ is monic iff h is monic in $\mathcal{C}$.

7

XM CCARD V1.0

For A object of $\mathcal{C}$, $h : f \to g$ of $\mathcal{C}/A$ is monic iff h is monic in $\mathcal{C}$.

7

There is a category containing an arrow that is epic and monic but not iso.

11

XM CCARD V1.0

There is a category containing an arrow that is epic and monic but not iso.

11

A category with all products and equalizers also has all pullbacks.

∞

XM CCARD V1.0

A category with all products and equalizers also has all pullbacks.

∞

If a category has a terminal object and pullbacks, it has binary products and equalizers.

ω

XM CCARD V1.0

If a category has a terminal object and pullbacks, it has binary products and equalizers.

ω

In $Set^{\mathcal{C}^{op}}$ limits are computed pointwise.

$\aleph_\lambda$

XM CCARD V1.0

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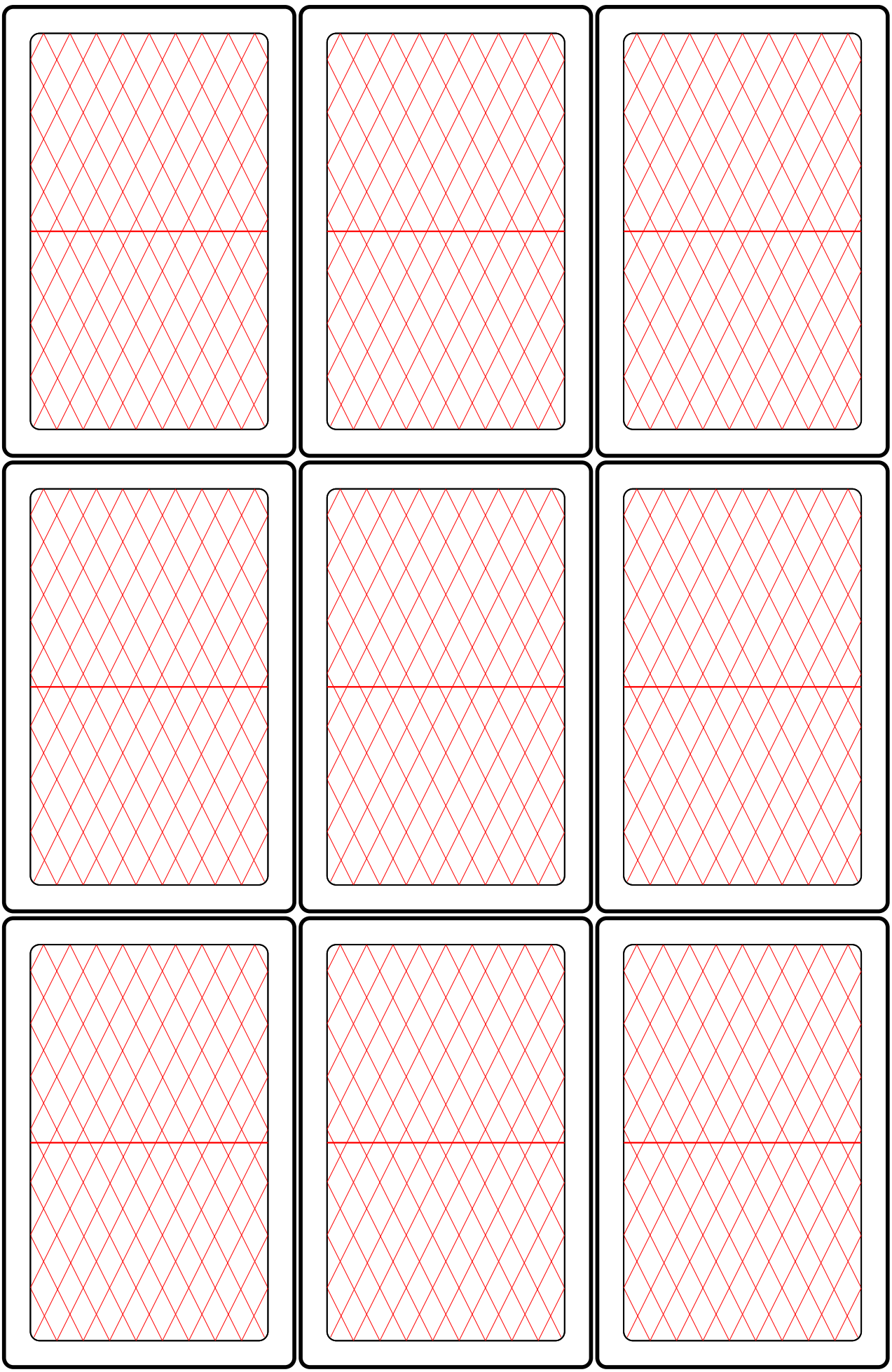

"It should be observed that the whole concept of a category is essentially an auxillary one."

(Eilenberg & Mac Lane)

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An embedding is a functor which is full and faithful and injective on objects.

2

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A natural transformation between two functors $F, G : \mathcal{C} \to \mathcal{D}$ is a family $(\mu_C : FC \Rightarrow GC)_{C \in \mathcal{C}_0}$.

3

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3

Objects that represent the same functor are isomporphic.

5

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5

Every functor preserves isomorphism.

7

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7

A faithful functor reflects epis and monos.

11

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11

A category is equivalent to a discrete category iff it is a groupoid and a preorder.

∞

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∞

A full and faithful functor reflects the property of being a terminal or initial object.

ω

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ω

If $G \circ F$ is an embedding, then so is F.

$\aleph_\lambda$

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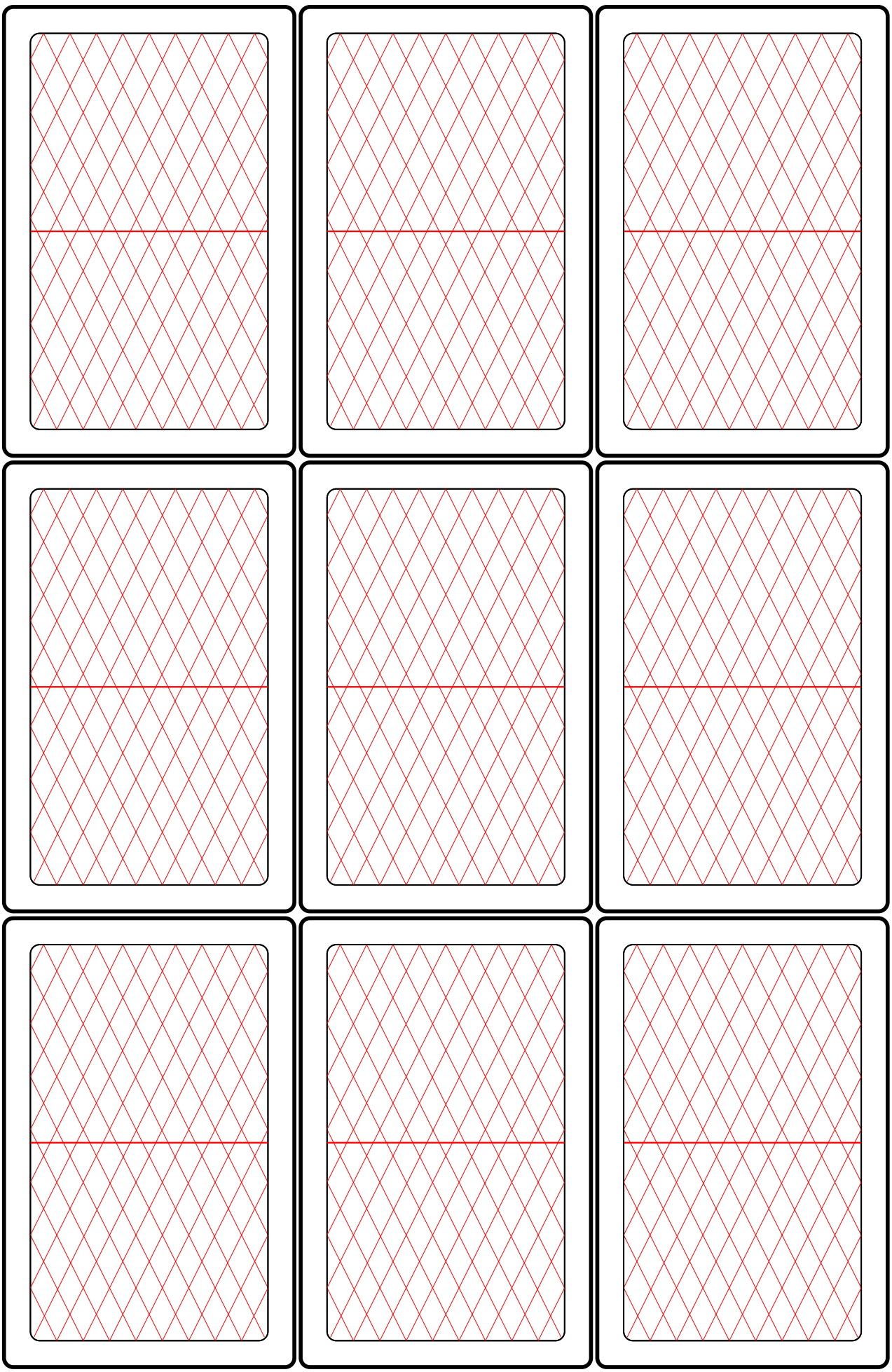

"A universal property is itself a reason to study it."

(Jaap van Oosten)

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For a category $\mathcal{C}$ and an object C, $\mathcal{C}/C$ is the slice category. Objects are arrows to C.

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2

The Yoneda Lemma is a generalization of Cayley's Theorem.

3

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3

There is a category with a terminal object and all products, but without exponentiation.

∞

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∞

Natural numbers object:

$$\begin{array}{ccccc} \mathbf{1} & \to & N & \to & N \\ & \searrow & \downarrow & & \downarrow \\ & & A & \to & A \end{array}$$

ω

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The functor $Y : \mathcal{C} \to \mathrm{Set}^{\mathcal{C}^{op}}$ is full and faithful.

5

The functor $Y : \mathcal{C} \to \mathrm{Set}^{\mathcal{C}^{op}}$ is full and faithful.

5

For each functor $T : \mathcal{C} \to \mathcal{C}$, the forgetful functor of **Alg** (T) creates limits.

7

For each functor $T : \mathcal{C} \to \mathcal{C}$, the forgetful functor of **Alg** (T) creates limits.

7

A subcategory $\mathcal{C}$ of $\mathcal{D}$ is full, if for all A, B: $\mathcal{C}(A,B) = \mathcal{D}(A,B)$.

11

A subcategory $\mathcal{C}$ of $\mathcal{D}$ is full, if for all A, B: $\mathcal{C}(A,B) = \mathcal{D}(A,B)$.

11

(Yoneda Lemma) For F of $Set^{\mathcal{C}^{op}}$, C of $\mathcal{C}$ there is a bijection $f_{C,F} : Set^{\mathcal{C}^{op}}(h_c, F) \to F(C)$.

$\aleph_\lambda$

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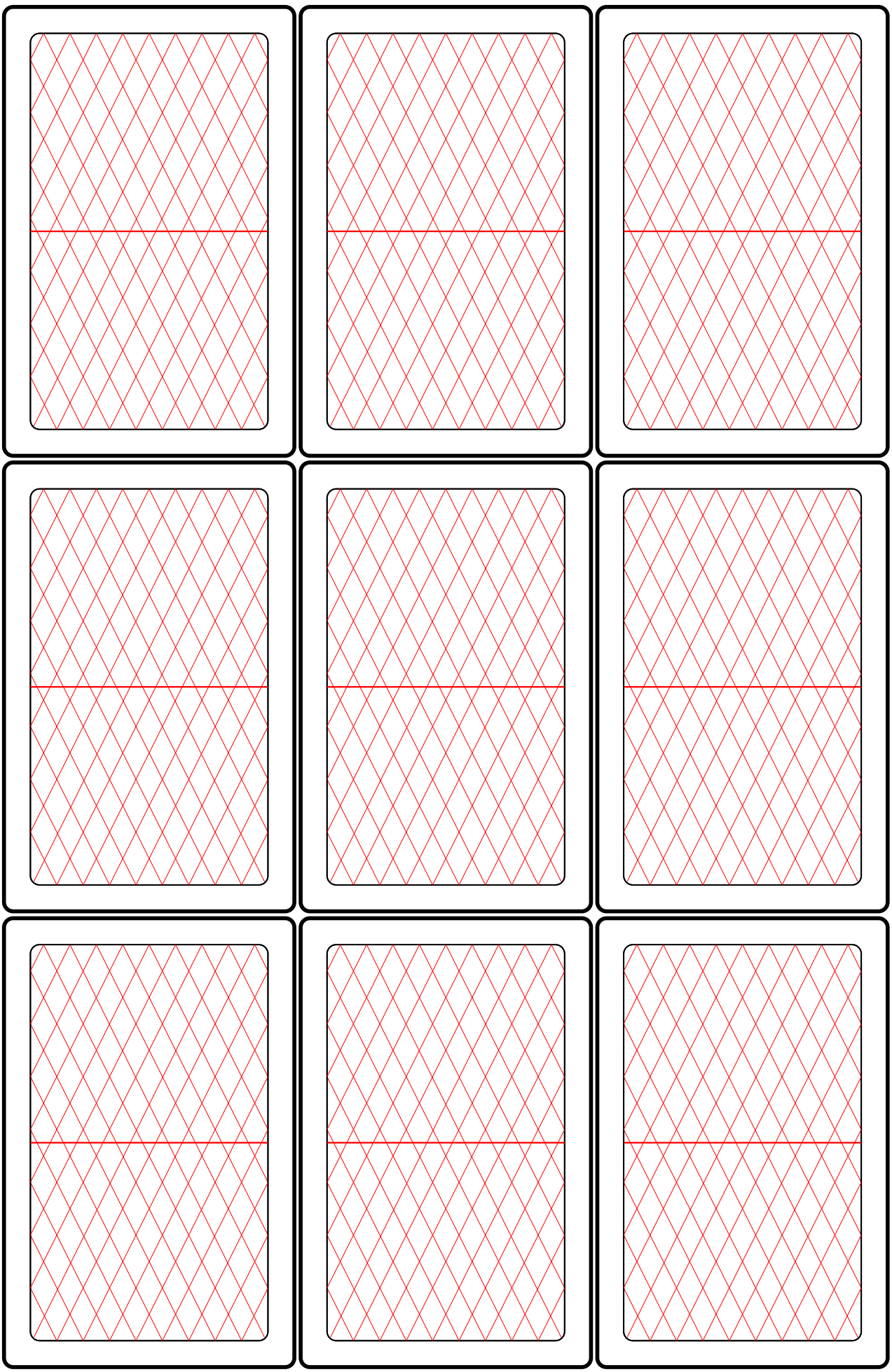
$\aleph_\lambda$



The slogan is: "Adjoint functors arise everywhere."

(Saunders Mac Lane)

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A simple example for adjoints is

$$\begin{array}{ccc} S & \xrightarrow{i} & U(List(S), \star, [\,]) \\ & {\scriptstyle 1}\searrow & \downarrow U(length) \\ & & U(\mathbb{N}, +, 0) \end{array}$$

2

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2

Any two left-adjoints of a functor are naturally isomorphic.

3

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3

For every adjunction there exists a counit ϵ.

5

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5

Galois connections are adjoint pairs.

7

Galois connections are adjoint pairs.

7

Let $F : \mathcal{C} \to \mathcal{D}$, $F \dashv G$ and $\overline{F} : \mathcal{D} \to \mathcal{E}$, $\overline{F} \dashv \overline{G}$. Then $\overline{F}F \dashv G\overline{G}$.

11

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11

In a category with finite products, $- \times A$ has a left adjoint iff A is a terminal object.

∞

In a category with finite products, $- \times A$ has a left adjoint iff A is a terminal object.

∞

If $G : \mathcal{C} \to \mathcal{C}$, $\mathcal{C}$ a poset and $G \dashv G$, then G is an isomorphism.

ω

If $G : \mathcal{C} \to \mathcal{C}$, $\mathcal{C}$ a poset and $G \dashv G$, then G is an isomorphism.

ω

The right adjoint to the binary product functor describes the exponential.

$\aleph_\lambda$

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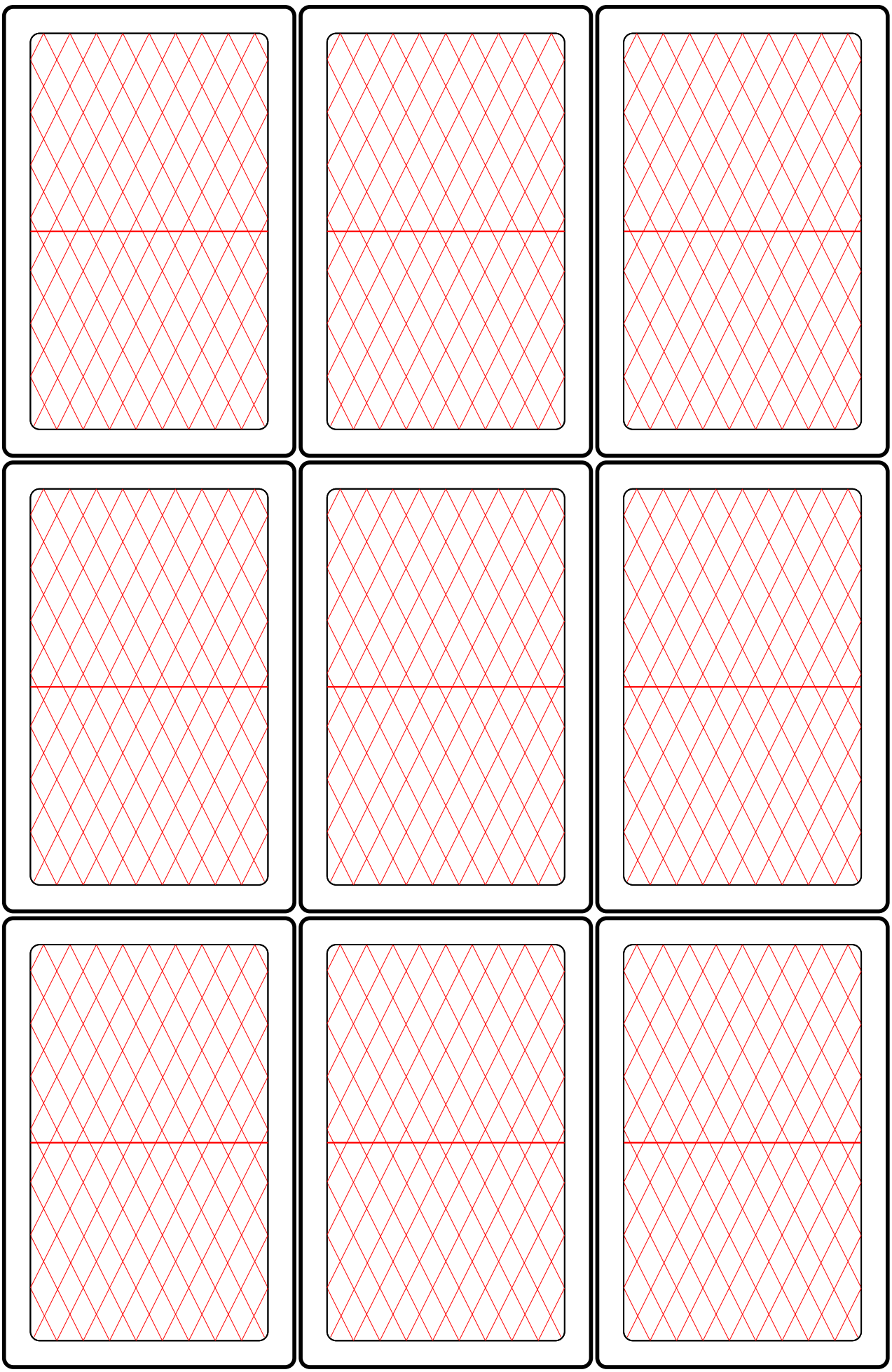

A cartesian closed category is a category with a terminal object, binary products and exponentiation.

∅

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A cartesian closed category is a category with a terminal object, binary products and exponentiation.

The category **Cat** is cartesian closed with $\mathcal{A}^{\mathcal{B}}$ the functor category.

2

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The category **Cat** is cartesian closed with $\mathcal{A}^{\mathcal{B}}$ the functor category.

In cartesian closed categories, coproducts distribute over products.

3

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In cartesian closed categories, coproducts distribute over products.

Every monotone map of X to a ω-cpo factors uniquely through the limits of this cpo.

5

5

Every monotone map of X to a ω-cpo factors uniquely through the limits of this cpo.

In a topos, all monics are equalizers and all monic epics are iso.

7

7

In a topos, all monics are equalizers and all monic epics are iso.

For toposes $\mathcal{E}$, $\mathcal{E}'$ also $\mathcal{E} \times \mathcal{E}'$ is a topos.

11

11

For toposes $\mathcal{E}$, $\mathcal{E}'$ also $\mathcal{E} \times \mathcal{E}'$ is a topos.

In a topos $\mathcal{E}$, one can interpret many-sorted predicate logic which also has constructors for subsets.

∞

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In a topos $\mathcal{E}$, one can interpret many-sorted predicate logic which also has constructors for subsets.

Theories in the simply typed λ-calculus correspond to cartesian closed categories.

ω

ω

Theories in the simply typed λ-calculus correspond to cartesian closed categories.

In a topos, take $x : \mathbf{1} \to A$, $w : A \to \Omega$. Then $x \in^{A} w$' iff $x \in w^{*}$.

$\aleph_\lambda$

$\aleph_\lambda$

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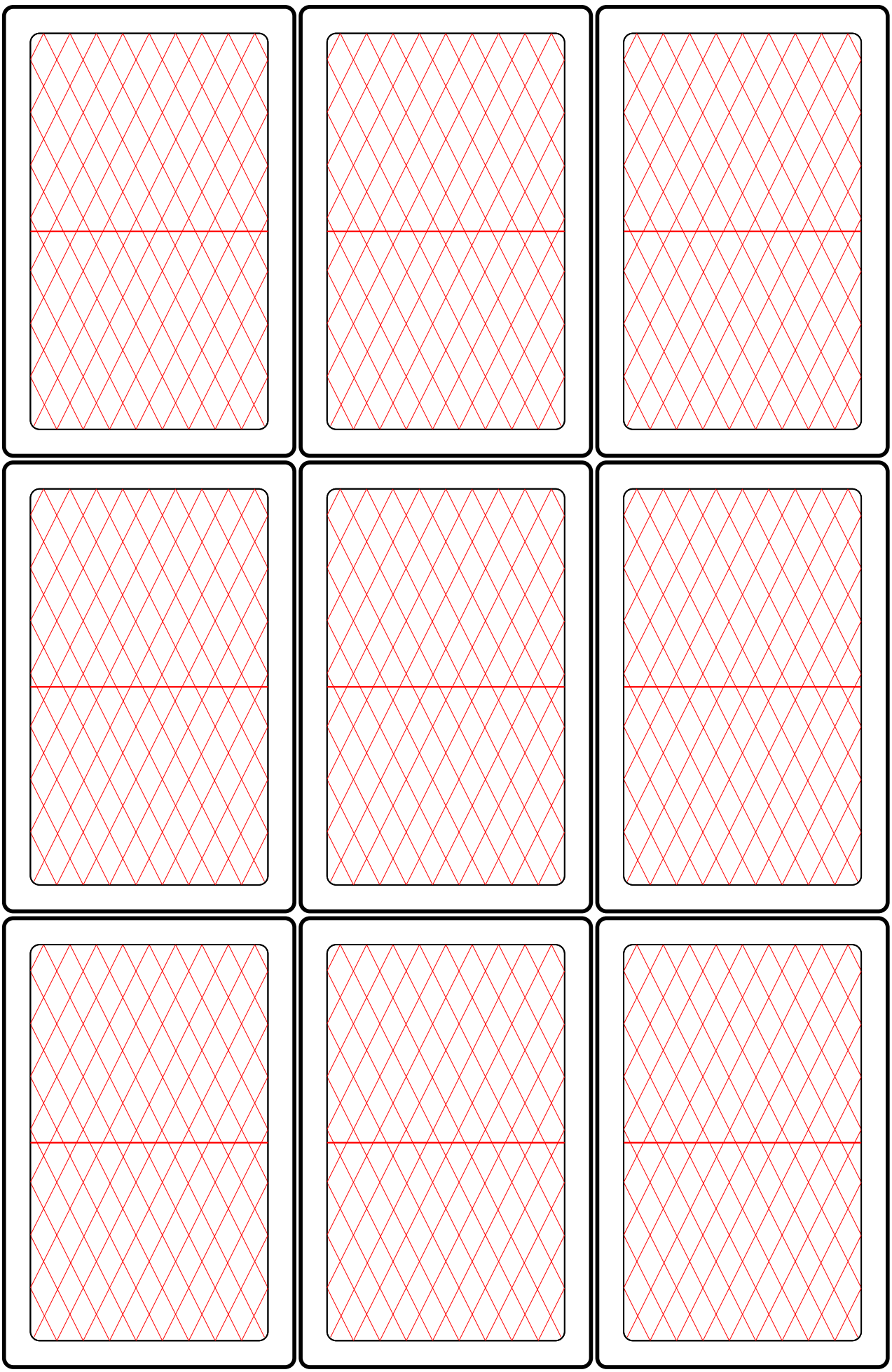

Each presheaf is a colimit of a representable presheaf.

3

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3

If there is an inital object $\mathbf{0}$ and an arrow $\mathbf{1} \to \mathbf{0}$ then $\mathbf{0}$ and $\mathbf{1}$ are zero objects.

2

If there is an inital object $\mathbf{0}$ and an arrow $\mathbf{1} \to \mathbf{0}$ then $\mathbf{0}$ and $\mathbf{1}$ are zero objects.

2

Equivalent subobjects have isomorphic domains.

$\varnothing$

Equivalent subobjects have isomorphic domains.

$\varnothing$

Each colimit is an extremal epi-sink.

11

Each colimit is an extremal epi-sink.

11

If $F : \mathcal{C} \to \mathcal{D}$ a full, faithful functor, $\forall\ f : FA \to FB\ \exists_1\ g : A \to B$ with $Fg = f$.

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If $F : \mathcal{C} \to \mathcal{D}$ a full, faithful functor, $\forall\ f : FA \to FB\ \exists_1\ g : A \to B$ with $Fg = f$.

7

If $F : \mathcal{A} \to \mathcal{B}$, B in $\mathcal{B}$ and a reflection along B exists, it is unique up to isomorphism.

5

If $F : \mathcal{A} \to \mathcal{B}$, B in $\mathcal{B}$ and a reflection along B exists, it is unique up to isomorphism.

5

IF $G : \mathcal{C} \to \mathcal{D}$ has a left adjoint F, F preserves all right Kahn extensions which exist in $\mathcal{C}$.

$\aleph_\lambda$

IF $G : \mathcal{C} \to \mathcal{D}$ has a left adjoint F, F preserves all right Kahn extensions which exist in $\mathcal{C}$.

$\aleph_\lambda$

Let $F : \mathcal{C} \to \mathcal{C}$ be a functor. If (A, f) is initial in the category $(F : \mathcal{C})$, then f is an isomorphism.

ω

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ω

For every monoidal $\mathcal{C}$, $A \in \mathcal{C}$ there is a unique morphism $\mathcal{D} \to \mathcal{C}$ of monoidal categories with $(-) \mapsto A$.

∞

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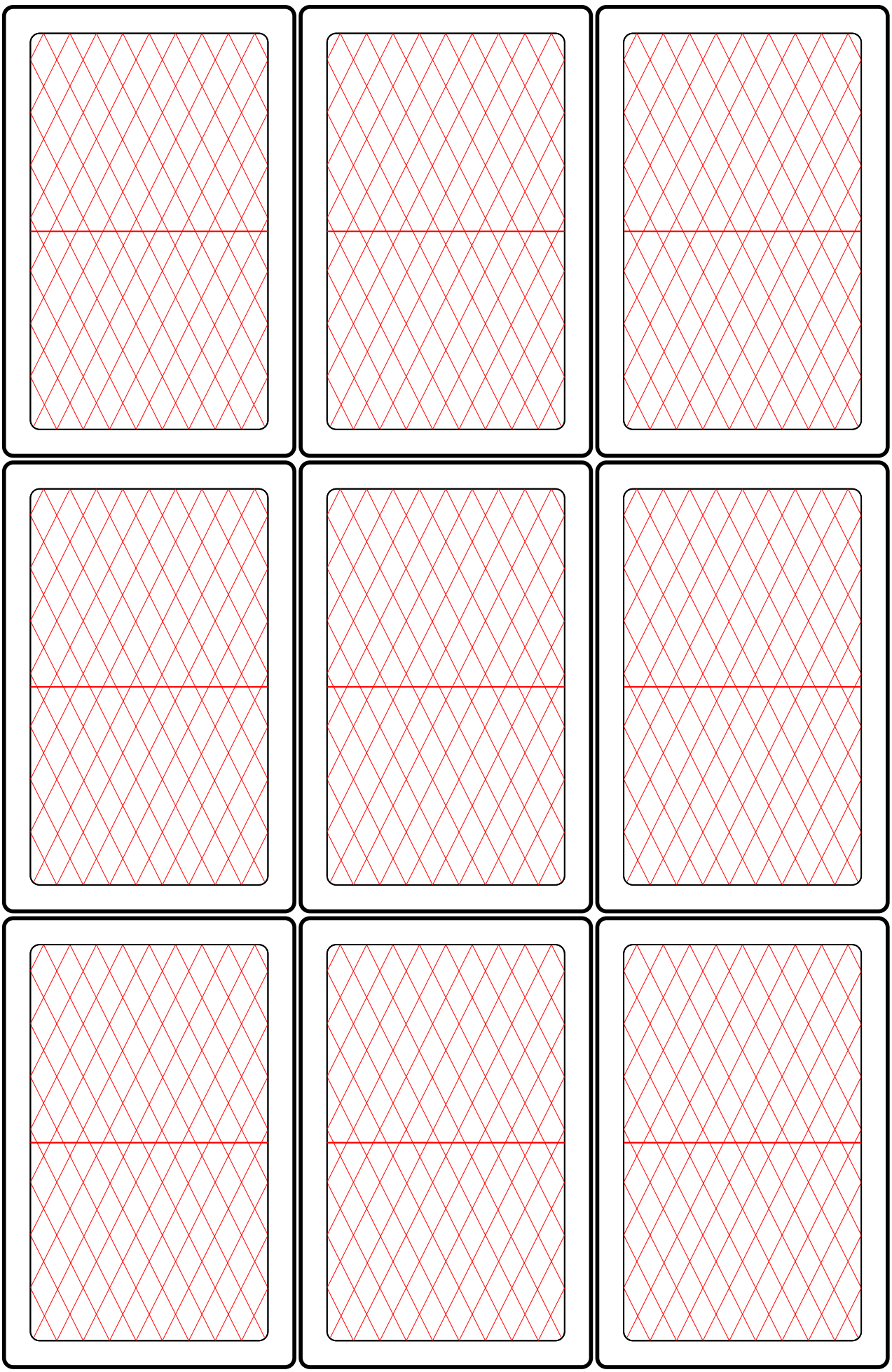
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